1) Radiation intensities (Watts / unit solid angle) of two different lossless antennas are given as:

\[
U_1 = \begin{cases} 
    B_1 \sin^2(\theta) \sin^3(\phi) & 0 \leq \theta \leq \pi, \quad 0 \leq \phi \leq \pi \\
    0 & \text{otherwise}
\end{cases}
\]

\[
U_2 = \begin{cases} 
    B_2 \sin^3(\theta) \sin^4(\phi) & 0 \leq \theta \leq \pi, \quad 0 \leq \phi \leq \pi \\
    0 & \text{otherwise}
\end{cases}
\]

Do (a) and (b) by using MATLAB:

a) Plot the radiation intensities at polar coordinates on x-y and y-z planes for both of the antennas. Normalize the plots as maximum value to be 1.

b) Sample the radiation intensities with \(\pi/20\)-radian steps in both theta and phi. With total 21x21 samples, determine the directivities (dimensionless) of both antennas numerically.

c) Find the directivities (dimensionless) analytically and compare the results with the ones found in (b).

d) Find \(B_1\) and \(B_2\) if the total radiated power is required to be 2 Watts for both of the antennas.

e) Find the azimuthal and elevation plane half-power beam widths in degrees.

2) Two lossless antennas have radiation intensities (Watts / unit solid angle):

\[
U_1 = \begin{cases} 
    2 \sin(\theta) \sin(\phi) \sin\left(\phi - \frac{\pi}{5}\right) \sin\left(\phi + \frac{\pi}{5}\right) & 0 \leq \theta \leq \pi, \quad 0 \leq \phi \leq \pi \\
    0 & \text{otherwise}
\end{cases}
\]

\[
U_2 = \begin{cases} 
    2 \sin(\theta) \sin(\phi) \sin\left(\phi - \frac{\pi}{7}\right) \sin\left(\phi + \frac{\pi}{7}\right) & 0 \leq \theta \leq \pi, \quad 0 \leq \phi \leq \pi \\
    0 & \text{otherwise}
\end{cases}
\]
Do (a), (b) and (c) using MATLAB.

a) Plot the radiation intensity with respect to phi on the x-y plane.

b) Find the azimuthal plane half-power beam width in degrees.

c) Find the ratio of the field intensities between the main and first side lobes (in dB).

d) Find the maximum radiation density in Watts/m² at a distance 1000 m. (Assume far-field).

3) An antenna having radiation intensity (Watts / unit solid angle)

\[
U_i = \begin{cases} 
B_i \cos^3(\theta) & 0 \leq \theta \leq \frac{\pi}{2}, \ 0 \leq \phi \leq 2\pi \\
0 & \text{otherwise}
\end{cases}
\]

is connected to a source having 2 V peak voltage and 50+75j ohms input impedance \((Z_g)\). The antenna has 50 ohms reactance \((X_A)\), 50 ohms radiation resistance \((R_r)\) and 25 ohms loss resistance \((R_L)\).

a) Find the power
   i) supplied by the generator
   ii) dissipated in the generator
   iii) delivered to the antenna
   iv) radiated by the antenna

b) Do (a) if the input impedance of the source is changed and matched to the antenna. Compare the results with the ones in (a).

c) Find \(B_1\) for both cases ((a) and (b)) and comment on the values.