HW 2

Note: Work independently. According to Bilkent University policy, the act of cheating is punishable by up to two semesters of suspension from the school.

1) A 300 MHz plane wave is given in phasor domain (+jwt notation) as:

\[ \mathbf{E}(\mathbf{r}) = \left[ \hat{x}(2 + 2\sqrt{3} j) + \hat{z}4e^{j\pi/6} \right] e^{-jky} \]

Assume free space propagation and \( k \) is the wave number.

a) At \( t = 0, 1 \times 10^{-9}, 2 \times 10^{-9} \) and \( 3 \times 10^{-9} \) seconds, plot the x component of the field with respect to y from 0 to 2 meters.

b) Do the same plots for z component of the field. What happens as the time goes on?

c) Assume that you are supplied only with the plots at \( t = 0 \) and \( 1 \times 10^{-9} \) seconds. How you can find the speed of the wave?

d) At \( y=0 \), plot both the x and z components of the field with respect to time. Make a guess about the polarization by using these plots. Explain.

e) Find the polarization by using the expression for the field.

f) Assume that we desire to change traveling direction to \(-y\). Write the new expression for the electric field, if the polarization will remain the same.

2) A transmitting antenna ‘A’ has electric field radiation as:

\[ \mathbf{E}(\mathbf{r}) = \left[ \hat{\theta} \sin(\theta) \cos(\phi) - \hat{\phi} \sin(\phi) \cos(\theta) \right] f(r, \theta, \phi) \]

where \( f \) is a scalar function.

a) Determine the direction of the linearly polarized receiving antenna, which minimizes the polarization loss on the y-z plane.

b) Determine the direction of the linearly polarized receiving antenna, which minimizes the polarization loss on the x-y plane.

c) Plot the polarization loss factors (PLF) on x-z plane with respect to theta for your receiving antennas in (a) and (b), when antenna ‘A’ is used as transmitter.

d) A circularly polarized plane-wave traveling in \(-x\) direction is received by the antenna A. Calculate the PLF (in dB) on the x axis, if the polarization is clockwise. Do the same, if the polarization is counter clockwise. What happens if the plane-wave is traveling in \(-y\) direction?
3) An antenna is known to have radiation intensity:

\[ U(\theta, \phi) = \begin{cases} 
U_0 \cos^3(\theta) & 0^\circ \leq \theta \leq \theta_0, 0^\circ \leq \phi \leq 2\pi \\
0 & \text{otherwise}
\end{cases} \]

a) Maximum effective aperture of the antenna is given as 0.25 m\(^2\) at 600 MHz. Find \( \theta_0 \).

b) What happens to effective aperture (find it) if the beam is adjusted as \( \theta_0 = 3^\circ \)?

4) The figures show two experiments, in which two antennas are used. In the first experiment, Antenna A (50 ohm) is used as the transmitter and connected to a source with 50-ohm terminal resistance. Antenna B (75 ohm) is used as the receiver and connected to a 25-ohm terminal resistance. In this experiment, the output power is found to be 1 Watt when the input power is 10 Watts. In the second experiment, the roles are exchanged and Antenna A becomes the receiver while the Antenna B becomes the transmitter.

**Experiment 1**

\[ Z_T^a = 50 \text{ ohm} \]

\[ Z_A^a = 50 \text{ ohm} \]

\[ V_{in} \]

\[ I_{sc} \]

\[ V_{oc} \]

\[ Z_T^b = 75 \text{ ohm} \]

\[ Z_A^b = 75 \text{ ohm} \]

\[ V_{out} \]

**Experiment 2**

\[ Z_T^a = 50 \text{ ohm} \]

\[ Z_A^a = 50 \text{ ohm} \]

\[ V_{out} \]

\[ V_{in} \]

\[ I_{sc} \]

\[ Z_T^b = 25 \text{ ohm} \]

\[ Z_A^b = 25 \text{ ohm} \]

a) Find the output power in the second experiment when the input power is again 10 Watts.

b) What should be the terminal resistance \((Z_T)\) of the transmitter while others remain the same in the second experiment so that the output power is 1 Watt, when the input power is again 10 Watts.

c) What should be the terminal resistance \((Z_T)\) of the receiver while others remain the same in the second experiment so that the output power is 1 Watt, when the input power is again 10 Watts.