System Transformation Induced by a Shift-Invariant Subspace with an Application to System Identification

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Biography:

Yoshito Ohta received the Dr.Eng. degree in Electronic Engineering from Osaka University in 1986. From 1986 to 1988, he was a visiting scientist at Laboratory for Information and Decision Systems, Massachusetts Institute of Technology. Since 2006 he has been a professor at Department of Applied Mathematics and Physics of Kyoto University. His research interests include modeling of control systems, networked control systems, and robust control. Dr. Ohta was an associate editor for IEEE Transactions on Automatic Control (2001-2005), and he was a member of the Board of Governors of the IEEE Control Systems Society (2008-2010). He is currently an associate editor of Automatica and European Journal of Control.

Abstract:

This paper studies a continuous to discrete-time conversion method based on a shift-invariant subspace in the signal space. Given an inner function, the corresponding shift-invariant subspace yields a decomposition of the signal space, which in turn induces a continuous to discrete-time conversion of linear time-invariant systems. The method can be extended to linear stochastic systems driven by a Wiener process. This transformation technique is useful in studying the H^∞ control problem, system approximation, and and system identification.

More specifically, let $L^2(j\mathbb{R})$ be the space of square integrable functions of frequency $j\omega \in j\mathbb{R}$ with the inner product

$$\langle u, v\rangle = \frac{1}{2\pi}\int_{-\infty}^{\infty} \overline{v(j\omega)}u(j\omega)\, d\omega.$$

The spaces H^2 and $H^2_\perp$ can be seen as subspaces of $L^2(j\mathbb{R})$ consisting of functions analytic in the open right-half and left-half planes, respectively. The space of bounded functions on the open right-half plane is denoted as H^∞. A function $\phi \in H^\infty$ is called inner if $|\phi(j\omega)| = 1$ for almost all ω. If ϕ is not a constant inner function, then the space ϕH^2 is a proper subspace of H^2. Hence its orthogonal complement $S = H^2 \ominus \phi H^2$ is a nonzero subspace in H^2. Furthermore, the signal spaces $L^2(j\mathbb{R})$, H^2, and $H^2_\perp$ admit the following decompositions:

$$L^2(j\mathbb{R}) = \oplus_{k=-\infty}^{\infty}\phi^k S, \quad H^2 = \oplus_{k=0}^{\infty}\phi^k S, \quad H^2_\perp = \oplus_{k=-\infty}^{-1}\phi^k S. \tag{1}$$

Thus a signal $u \in L^2(j\mathbb{R})$ is written as

$$u = \sum_{k=-\infty}^{\infty} \phi^k u_k, \quad u_k \in S. \tag{2}$$

Moreover, $\|u\|^2 = \sum_{k=-\infty}^{\infty} \|u_k\|^2$. In this sense, we can identify $L^2(j\mathbb{R})$ and $\ell^2(S)$.

Consider the transfer function $h(s) = D + C\left(sI - A\right)^{-1} B$, where A does not have eigenvalues on the imaginary axis. Then $h(s)$ is a bounded operator on $L^2(j\mathbb{R})$. Let u, $y \in L^2(j\mathbb{R})$ be the input and the output of the transfer function $h(s)$. Then the isomorphism between $L^2(j\mathbb{R})$ and $\ell^2(S)$ induces a bounded map h_D by the commutative diagram:

$$\begin{array}{ccc} L^2(j\mathbb{R}) & \xrightarrow{h} & L^2(j\mathbb{R}) \\ \downarrow & & \downarrow \\ \ell^2(S) & \xrightarrow{h_D} & \ell^2(S) \end{array} \tag{3}$$

Because of the structure (1), the map h_D has a time-invariant discrete-time state equation:

$$\xi_{t+1} = \mathbf{A}\xi_t + \mathbf{B}u_t, \quad y_t = \mathbf{C}\xi_t + \mathbf{D}u_t, \tag{4}$$

where the operators $\mathbf{A} : \mathbb{R}^n \to \mathbb{R}^n$, $\mathbf{B} : S \to \mathbb{R}^n$, $\mathbf{C} : \mathbb{R}^n \to S$, and $\mathbf{D} : S \to S$ satisfy

$$\mathbf{A}\xi = \phi^{\sim}(A)\xi, \tag{5}$$

$$\mathbf{B}u = \frac{1}{2\pi}\int_{-\infty}^{\infty} \overline{\left(\phi^{\sim}(A)\left(j\omega I + A\right)^{-1} B - \phi(j\omega)\left(j\omega I + A\right)^{-1} B\right)} u(j\omega) d\omega, \tag{6}$$

$$\left(\mathbf{C}\xi\right)(s) = \left(C\left(sI - A\right)^{-1} - \phi(s)C\left(sI - A\right)^{-1}\phi^{\sim}(A)\right)\xi, \tag{7}$$

$$\left(\mathbf{D}u\right)(s) = h(s)u(s) - \phi(s)C\left(sI - A\right)^{-1}\mathbf{B}u. \tag{8}$$

For example, if $\phi(s) = e^{-sT}$, $T > 0$, then the orthogonal complement S of the shift invariant subspace ϕH^2 is equal to the image of $L^2(0, T)$ by the Fourier transform. Then the transformed system (4) is nothing but the lifted system studied in the context of sampled-data control [1, 2]. In [3, 4], the transformation when $\phi(s)$ is rational and $h(s)$ is stable is studied using an appropriate basis introduced to S. In particular, if $\phi(s) = (p - s)/(p + s)$, $p > 0$, then the space S is one dimensional spanned by $\{1/(p + s)\}$, and the system (4) is given by the linear fractional transformation. The transformation formula (5)-(8) is a generalization in the sense that it does not assume that S is finite dimensional and that the system matrix A mat have unstable eigenvalues. Note that [5] considered a similar formula for a restricted class of systems. A detailed derivation of (5)-(8) is discussed in [6].

Applications of the transformation are seen in the area of the H^∞ control problem, system approximation, and system identification. In [5], Schmidt pairs of the Hankel operator for a class of infinite dimensional systems are characterized, and then the H^∞ sensitivity minimization problem and the balanced and truncation method are investigated. In [7], the transformation for stochastic systems is applied to the continuous-time subspace identification problem. Furthermore, a recursive identification method is proposed in [8], and a closed loop identification method is studied in [9]. In this paper, we describe various techniques for system identification using the transformation method.