The o/p time constant is $RC$ which creates a
3dB cut-off frequency $= w_c = \frac{1}{2\pi RC} = \frac{1}{2\pi C}$
for $L = 0$. The question is:

Can we extend the BW of an amplifier ($w_c$) by adding an inductor $L$ with a suitable value in series with the drain resistor $R$. In this proposed solution, we are creating a parallel resonant circuit with a lossy inductor driven by a current source.
The circuit is equivalent to

\[ R_p = \frac{(Q^2+1)}{Q^2} R \]

\[ L_p = \frac{Q^2+1}{Q^2} L_s \] & \[ Q = \frac{wL}{R} \]

which is an exact expression at any \( w \) and only at that \( w \).

If we now draw the phasor diagrams for frequencies at which \( R, \frac{1}{\omega C} \) & \( wL \) are not very much different from each other:

![Phasor Diagrams](image)

At frequencies approaching \( w_1 \) or going beyond it (i.e. the value \( \frac{1}{\omega C} \) being not very much different from \( R \)) a value of \( L \) can be chosen such that \( JB_1 \) compensates \( j\omega C \) to some extent, which results at a value of \( G_1 < \frac{1}{R^2} \) (because \( Q > 0 \)) and therefore creating an overall impedance greater than the impedance of the output load without \( L \) (maybe perhaps greater than \( R \)). Therefore the response in the vicinity of \( w_1 \) is increased.
This is a low $Q$ circuit, therefore exact derivation is required.

\[
Z(j\omega) = \frac{(R + j\omega L)}{1 + j\omega CR} = \frac{R + j\omega L}{1 + j\omega CR - \frac{1}{j\omega C}} = R \left( \frac{1 + j\omega \frac{L}{R}}{1 - \frac{L^2 C}{R}} \right)
\]

\[
\frac{Z(j\omega)}{R} = \frac{1 + j\omega \left( \frac{L}{R} \right)}{1 - \frac{L^2 C}{R} + j\omega CR}
\]

Let $\tau = \frac{L}{R}$, $\tilde{\tau} = RC = \frac{1}{\omega_1}$

\[
m = \frac{\tilde{\tau}}{\tau} = \frac{RC}{L/R} = \frac{1}{\omega_1} \frac{\tau}{L} = \frac{R^2 C}{L}
\]

\[
\Rightarrow L = \tau R, \quad C = \frac{m^2 \tau}{R} \Rightarrow LC = \frac{\tau R \frac{m^2 \tau}{R}}{m^2} = m \tau^2 = m \frac{1}{\omega_1^2}
\]

and $\tau = \frac{1}{m \omega_1} = \frac{L}{R}$

\[
\Rightarrow \left| \frac{Z(j\omega)}{R} \right| \text{ becomes}
\]

\[
\left| \frac{Z(j\omega)}{R} \right| = \left[ \frac{1 + (\omega \tau)^2}{(1 - \omega^2 \tau^2 m^2)^2 + (\omega^2 \tau^2 m^2)^2} \right]^{1/2}
\]

or by defining $m_n = \frac{w}{m \omega_1}$

\[
\frac{Z(j\omega)}{R} = \left[ \frac{1 + m_n^2 w_n^2}{(1 - m_n^2 w_n^2)^2 + w_n^2} \right]^{1/2}
\]

by defining this ratio as $G_n(j\omega)$
We are now trying to find the new 3 dB point with the introduced $L$. Therefore we can find the bandwidth extension ratio $\omega_n$ by equating $|G_n(j\omega_n)|$ to $\frac{1}{V_2^n}$ or $|G_n(j\omega_n)|^2 = \frac{1}{2}$ and solving for $\omega_n$ taking $m$ as the parameter defining the ratio of time constants

$$m = \frac{T_c}{T} = \frac{\omega_n}{\omega_1}$$

where $\omega_1 = \frac{1}{RC}$ & $\omega_L = \frac{1}{R}$ or the ratio of the cut-off frequencies.

$$\frac{1}{2} = \frac{1 + \frac{\omega_{n3}^2}{m^2}}{(1 - \frac{\omega_{n3}^2}{m^2})^2 + \omega_{n3}^2}$$

where $\omega_{n3}$ is the 3 dB point of the normalized frequency or the bandwidth extension ratio.

letting $d = \omega_{n3}^2$

$$\frac{1}{2} = \frac{1 + \frac{d}{m^2}}{(1 - \frac{d}{m^2})^2 + d} \quad \Rightarrow 1 - 2 \frac{d}{m} + \frac{d^2}{m^2} + d = 2 + \frac{2d}{m^2}$$

$$\frac{d^2}{m^2} - d \left[1 - \frac{2}{m} - \frac{2}{m^2} \right] \frac{1}{\omega_c} = 0$$

$$\omega_{n3} = \sqrt{\frac{(m^2 - 2m - 2)^2 + 4m^2}{4}}$$
\[
\frac{w_{3\text{db}}}{w_1} = w_{3\text{db}} = \left[ -\frac{m^2}{Z} + m + 1 + \sqrt{\left(\frac{m^2}{Z} + m + 1\right)^2 + m^2} \right]^{1/2}
\]

where \(w_{3\text{db}}\) is the 3 dB radial frequency. We take only the positive quadratic since the negative one results at a negative frequency.

For different values of the index \(m\), we get different response curves which can approximately be plotted as:

- \(m = 2.41\) (maximally flat)
- \(m = 2\)
- \(m = 1.41\) (maximally flat)
- \(m = 3.1\) (Best group delay)
- \(m = \infty\) (without an inductor)
Series Peaking

Gain $G(j\omega) = \frac{Z_{in}(j\omega) \cdot \frac{Z_c}{Z_c + Z_L}}{Z_{out}} = G(j\omega)$

\[
G(j\omega) = \frac{1}{(1 - \omega^2 L_2 C_L) + j\omega R_L C_L}
\]

Letting $\omega_1 = \frac{1}{R_L C_L}$, $m = \frac{T_c}{\tau} = \frac{R_C}{L/R}$

$\tau = \frac{L_2}{R_L}$

$T_c = R_L C_L = \frac{1}{\omega_1}$ again

$L_2 C_L = \frac{1}{m \omega_1^2}$ and $R_L C_L = \frac{1}{\omega_1}$ are obtained
defining \( G_n(j\omega) = \frac{G(j\omega)}{R_n} \), it becomes

\[
G_n(j\omega) = \frac{1}{\left(1 - \frac{w^2}{w_1^2 m} \right) + j\omega \frac{w_n}{w_1}} = \frac{1}{\left(1 - \frac{w_n^2}{m} \right) + j\omega n}
\]

where \( w_n = \frac{w}{w_1} \)

\[
|G_n(j\omega)|^2 = \frac{1}{\left(1 - \frac{w_n^2}{m} \right)^2 + w_n^2} = \frac{1}{1 - 2\frac{w_n^2}{m} + \frac{w_n^4}{m^2} + w_n^2}
\]

at the 3 dB point \( |G_n(j\omega)|^2 = \frac{1}{2} \Rightarrow \)

\[
\frac{1}{2} = 1 - w_n^2 \left(1 - \frac{m}{m} \right) + \frac{w_n^4}{m^2}
\]

letting \( w_n^2 = d \)

\[
\frac{1}{m^2} d^2 + \left(1 - \frac{2}{m} \right) d - 1 = 0 \Rightarrow d^2 + d(m^2 - 2m) - m^2 = 0
\]

\[
a = 1 \quad b = m^2 - 2m \quad c = -m^2
\]

\[
d_{1,2} = -\frac{b}{2a} \pm \frac{\sqrt{b^2 - \frac{\Delta}{a}}}{a} = \frac{2m - m^2}{2} \pm \frac{(2m - m^2)^2}{4} + m^2
\]

\[
taking only the positive quadratic again as \( w_n \) should be a real number (negative quadratic results in imaginary \( w_n \))
\]

\[
w_n^2 = d = m - \frac{m^2}{2} + \frac{\sqrt{m^2(2 - m)^2 + 4m^2}}{4}
\]

\[
= m - \frac{m^2}{2} + \left( \frac{4m^2 - 4m^3 + m^4 + 4m^2}{4} \right)^{\frac{1}{2}}
\]

\[
w_n = d^{\frac{1}{2}} = \left[ m - \frac{m^2}{2} + \left[ \frac{8m^2 - 4m^3 + m^4}{4} \right]^{\frac{1}{2}} \right]^{\frac{1}{2}}
\]
\[ w_{n3} = \left[ m - \frac{m^2}{2} + \left[ 2 m^2 - m^3 + \frac{m^4}{4} \right]^{1/2} \right]^{1/2} \]

for \( m = 2 \) \( \Rightarrow w_{n3} = \sqrt{2} \)

for \( m = 3 \) \( \Rightarrow w_{n3} = 1.36115 \)
There are other peaking arrangements.

Amplifier with shunt & series peaking

Shunt and double-series peaking exchanges increased delay with bandwidth

\[ L = \frac{R_c^2 C_L}{2(1+k)} \]

\[ k = \frac{1}{3} \] Butte north - type maximally flat supre

\[ k = \frac{1}{2} \] maximally flat group delay

Amplifier with T-coil BW enhancement