Tapped Impedance match

\[ \text{Vin} \quad 3X_{c_1} \quad 3X_{c_2} \quad R_2 \quad V_0 \]

\[ \text{Rin} \quad \text{If the is very high } \Rightarrow R_2 \gg (X_{c_1} \Rightarrow) \]
\[ \text{this is a simple voltage divider and as the result} \]
\[ w_0 = n \text{Vin} = \frac{X_{c_2}}{X_{c_1} + X_{c_2}} \quad \Rightarrow \quad n = \frac{X_{c_2}}{X_{c_1} + X_{c_2}} = \frac{\frac{1}{X_{c_2}}}{\frac{1}{X_{c_1}} + \frac{1}{X_{c_2}}} \]

\[ \Rightarrow n = \frac{1}{C_2} \frac{C_1 C_2}{C_1 + C_2} = \frac{C_1}{C_1 + C_2} \]

Since dissipation must be the same the equivalent resistance Rin is related R2 as

\[ \frac{V_{in}^2}{R_{in}} = 1 \Rightarrow \frac{V_{in}^2}{V_o^2} = \frac{R_{in}}{R_2} = n^2 \]

\[ \Rightarrow R_{in} = n^2 R_2 \quad \text{as in a transformer} \]

Note that the loss is always transformed, but to observe it purely you need to turn out the capacitive part.
The capacitive part to be tuned out is given by approximately again

\[ X_c_1 + X_c_2 \text{ as } R_2 \ll X_c_2 \Rightarrow \]

\[ C_{eq} = C_1 \text{ in series with } C_2 = \frac{C_1 C_2}{C_1 + C_2} \]

To solve the problem exactly at resonance please refer to "a note on impedance transformer".

The inductive divider acts similarly with only the capacitive and inductive impedances interchanged.
A note on resonant impedance transformer

In this circuit we would like to find the impedance seen in parallel with the inductor, namely \( R_{in} \). The problem is similar to T-match, with the differences:

a) The ground is at a different position
b) We are trying to find the equivalent resistance seen in parallel with \( L \) instead of in parallel with \( C_1 \)

The above circuit is equivalent to grounding changed

\[
\begin{align*}
X_L & \quad X_{C_1} = C_1 \\
X_{C_{25}} & = C_2 \\
R_{E} & = R_L / (Q_R^2 + 1) \\
X_{C_{25}} & = \frac{Q_R^2 R_L}{Q_R^2 + 1} X_C
\end{align*}
\]

The circuit resonates at the frequency \( \omega \) defined by \( X_L = X_{C_1} + X_{C_{25}} \)
Our aim is to transform the resistance $R_L$ to $R_{in}$ at resonance. If we let

$$K = \frac{R_{pl}}{R_2} = \frac{R_{in}}{R_2}$$

where $R_{pl}$ is the equivalent resistance in parallel with $L$.

$$R_{in} = R_{pl} = R_1(Q^2+1)$$

where $Q = Q_L + Q_R$

$$(Q^2+1)R_1 = R_{in} Q$$

$$(Q_R^2+1)R_1 = R_2$$

dividing the 2 equations

$$\frac{Q^2+1}{Q_R^2+1} = \frac{R_{in}}{R_2} = K$$

$$Q^2 + 1 = KQ_R^2 + K \Rightarrow KQ_R^2 = Q^2 + 1 - K$$

$$Q_R = \sqrt{\frac{Q^2 + 1 - K}{K}}$$

$$Q_L = Q - Q_R = Q - \sqrt{\frac{Q^2 + 1 - K}{K}}$$

After finding $Q_L$ & $Q_R$ for the required $Q$ & $K$ $R_{in}$ can be found by

$$R_{in} = \frac{R_2}{Q_R^2 + 1}$$

$X_{c_1} = \frac{Q_LR_1}{R_{in}}$ and $Q = \frac{R_{in}}{X_L}$

$$\Rightarrow X_{c_1} = Q_L R_1$$ and $X_L = \frac{R_{in}}{Q}$
or alternatively use the following steps in solving the problem:

1.) \((Q^2 + 1) R_T = R_{in} \Rightarrow R_T = \frac{R_{in}}{Q^2 + 1}\) and \(X_L = \frac{R_{in}}{Q}\)

2.) \((Q_R^2 + 1) R_T = R_2 \Rightarrow Q_R^2 = \frac{R_2}{R_T} - 1 \Rightarrow |X_{cd}| = \frac{R_2}{Q_R}\)

3.) \(Q - Q_R = Q_L = \frac{|X_{cd}|}{R_T}\)

Example:

Find the reactances of the upward transformer which converts 500V into 1000V employing a \(Q = 3\).

\(R_{in} = 1000\Omega\)

\((Q^2 + 1) R_T = R_{in}\)
\((3^2 + 1) R_T = 1000\)
\(\Rightarrow R_T = \frac{1000}{10} = 100\Omega\)

\((Q_R^2 + 1) R_T = 500\Omega \Rightarrow\)

\(Q_R^2 + 1 = \frac{500}{100} = 5 \Rightarrow Q_R = 2\)

\(\frac{|X_{cd}|}{100} = 3 - 2 = Q_L = 1 \Rightarrow X_{cd} = -100\)

\(Q_R = 2 \Rightarrow X_{cd} = \frac{-500}{2} = -250\) j

\(X_L = \frac{R_{in}}{Q} = \frac{1000}{3} = 333.33\)