Chapter 2 : CIRCUIT THEORY PRIMER

Electronics is not an abstract subject. Electronics is about designing and constructing instruments to fulfill a particular function, and be helpful to mankind. Electrical energy, voltage and current are all measurable, physical phenomena. Circuits are made up of resistors, capacitors, inductors, semiconductor devices, integrated circuits, energy sources, etc., in electronics. We design circuits employing these components to process electrical energy to perform a particular function. Circuits may contain a very large number of components. TRC-10 is an instrument with modest component count, yet it has about 200 components. If there should not be a set of rules, which tell how these components must be used, electronics would have never been possible. Algebra and differential equations are the tools that are used to both define the functions of elements and their inter-relations. The mathematics of circuit analysis and synthesis, models and set of rules developed for this purpose is altogether called *circuit theory*.

2.1. Energy sources

All circuits consume energy in order to work. Energy sources are either in form of *voltage sources* or *current sources* in electronic circuits. For example, batteries are voltage sources.

The voltage and current source symbols are shown in Figure 2.1. We need the concept of *ideal source* in order to model the real sources mathematically. An ideal voltage source is capable of providing the defined voltage across its terminals regardless of the amount of current drawn from it. This means that even if we short circuit the terminals of a voltage source and hence draw infinite amperes of current from it, the ideal voltage source, e.g. the one in Figure 2.1(a), will keep on supplying V_o to the short circuit. This inconsistent combination obviously means that the ideal supply is capable of providing infinite amount of energy. Similarly an ideal current source can provide the set current whatever the voltage across its terminals may be.

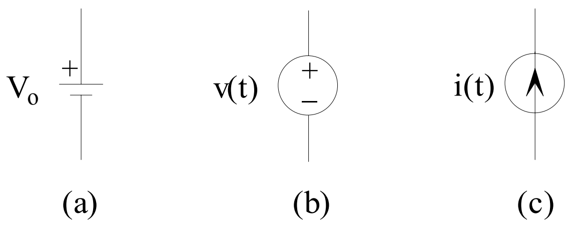


Figure 2.1 (a) d.c. voltage source, (b) general voltage source, (c) current source

Energy sources of infinite capacity are not available in nature. Should that be possible, then we should not worry about energy shortages, or on the contrary, we should be worrying a lot more on issues like global warming. The concept of ideal source, however, is very important and instrumental in the analysis of circuits.

Real sources deviate from ideal sources in only one aspect. The voltage supplied by a real source has a dependence on the amount of current drawn from it. For example, a battery has an internal resistance, and when connected to a circuit, its terminal voltage decreases by an amount proportional to the current drawn from it. This is depicted in Figure 2.2(a).

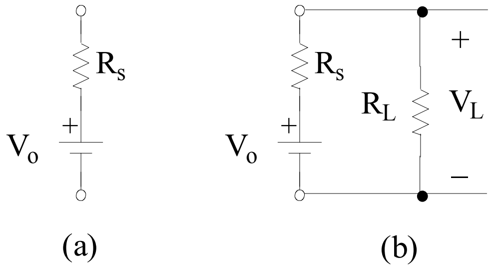


Figure 2.2 (a) Equivalent circuit of a battery, (b) a battery circuit

The internal resistance of the battery is denoted as R_s , in Figure 2.2. When there is no current drawn from the battery, the voltage across the terminals is V_o. When a *load* resistance R_L, such as a flash light bulb, is connected to this battery, the voltage across the battery terminals is no longer V_o. This circuit is given in Figure 2.2(b).

Ohm's Law says that the voltage across a resistor is proportional to the current that passes through it, and the proportionality constant is its *resistance*, R:

$$R = \frac{V}{I}.$$

R is measured in *ohms*, denoted by Ω (Greek letter omega), such that 1V/1A=1Ω. Inverse of R is called *conductance* and denoted by G and measured in terms of *siemens*, S:

$$G = \frac{I}{V} = \frac{1}{R}.$$

1/(1Ω) is 1S. Kirchhoff's Voltage Law (KVL) states that, when the resistors are connected in series, the total voltage across all the resistors is equal to the sum of voltages across each resistor and the current through all the resistors is the same. Therefore in a series connection the total resistance is equal to the sum of the resistances.

In the circuit in Figure 2.2(b) we have R_s and R_L connected in *series*. Hence the total resistance that appears across V_o (the ideal source voltage) is

$R_T = R_s + R_L$

and the current through them is

$$I = \frac{V_o}{R_s + R_L}.$$

The voltage across the battery terminals (or R_L) V_L is IR_L or $[V_o/(R_s+ R_L)]\ R_L$. This value is also equal to $V_o-[V_o/(R_s+ R_L)]\ R_s$. Therefore the terminal voltage of a battery is less than its nominal value when loaded by a resistance. Note that here we *modeled* a real source by a combination of an ideal voltage source, V_o, and a resistor, R_s.

The power drawn from the battery is

$$P_b = IV_o = \frac{V_o^2}{R_s + R_L}$$

where as the power delivered to the load, R_L is

$$P_L = IV_L = \frac{V_o^2 R_L}{(R_s + R_L)^2}.$$

The difference between P_b and P_L, $P_b[1- R_L /(R_s+ R_L)]$ is consumed by the internal resistor, R_s. Note that as R_s gets smaller, this power difference tends to zero.

Supplies in TRC-10 are not batteries. There are +15V, -15V and a +8V dc voltage supplies in TRC-10, all of which are obtained from mains by *a.c. to d.c conversion* and *voltage regulation*. Such voltage supplies behave differently compared to batteries. There is a specified output current limit for this kind of real voltage supplies. When the current drawn from the supply is below this limit, the terminal voltage of the supply is almost exactly equal to its nominal no-load level. When the limit is exceeded the terminal voltage drops abruptly. The regulated supplies behave almost like ideal supplies as long as the drawn current does not exceed the specified limit.

2.2. Resistors

Resistors dissipate energy. This means that they convert all the electrical energy applied on them into heat energy. As we increase the power delivered to a resistor it warms up.

When resistors are connected in parallel, the voltage, V, across every one of them will be the same, but each one will have a different current passing through it:

$$I_i = \frac{V}{R_i}.$$

Figure 2.3 depicts parallel-connected resistors.

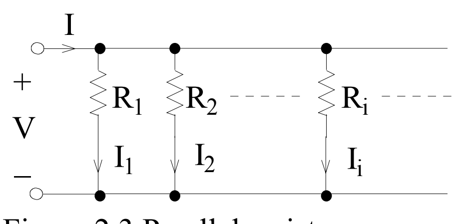


Figure 2.3 Parallel resistors

Kirchhoff's Current Law (KCL) says that the total current that flow into a node is equal to the total current that flow out of a node. The total current that flow into this parallel combination is I. Hence

$$I = \sum_i I_i$$

But $I_i = V/R_i$ and total resistance of the parallel combination is

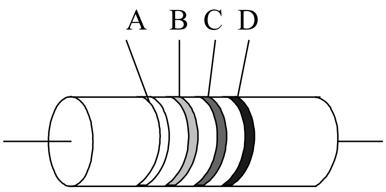
$$R_T = \frac{V}{I} = \left(\frac{1}{R_1} + \frac{1}{R_2} + ... + \frac{1}{R_i} + ... \right)^{-1}$$

or

$$G_T = \frac{I}{V} = G_1 + G_2 + ... + G_i + ...$$

We denote parallel combination of resistors as R_1// R_2 when R_1 and R_2 are in parallel and as R_1//R_2//R_3 when R_1, R_2 and R_3 are in parallel.

The resistors that we use in electronics are made of various materials. Most abundant are carbon resistors. There is a color code for resistance values. The resistance of a resistor is expressed in terms of a sequence of colored bands on the resistance. The color code is given in Figure 2.4.



A: First significant figure of resistance
B: Second significant figure
C: Multiplier
D: Tolerance

Color	Significant figure	Multiplier	Tolerance
Black	0	E0	
Brown	1	E1	
Red	2	E2	
Orange	3	E3	
Yellow	4	E4	
Green	5	E5	
Blue	6	E6	
Violet	7	E7	
Gray	8	E8	
White	9	E9	
Gold			%5
Silver			%10
No color			%20

Figure 2.4 Resistor color codes

Most of the common resistors are available in standard values. The two significant figures of standard resistor values are:

10, 12, 15, 18, 22, 27, 33, 39, 47, 56, 68, and 82.

Hence, a 100 Ω resistor is marked as brown-black-brown and a 4.7 KΩ resistor is marked as yellow-violet-red.

2.2.1. Resistive circuits

Electrical circuits can have resistors connected in all possible configurations. Consider for example the circuit given in Figure 2.5(a). Two resistors are connected in series, which are then connected in parallel to a third resistor, in this circuit.

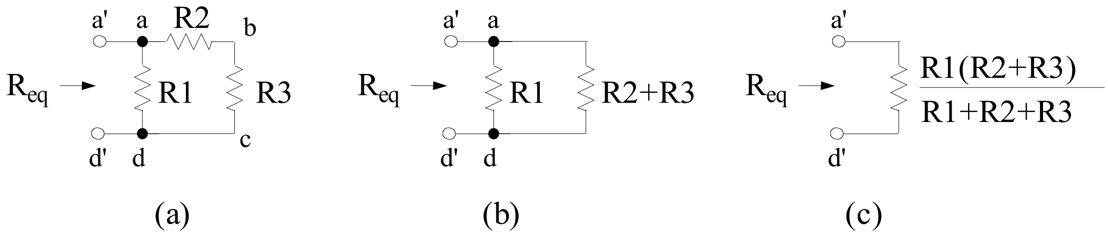


Figure 2.5 (a) Resistive circuit, (b) series branch reduced to a single resistance and (c) equivalent resistance.

In order to determine the overall equivalent resistance R_{eq}, which appears across the terminal *a'-d'*, we must first find the total resistance of *a-b-c* branch. R2 and R3 are connected *in series* in this branch. The total branch resistance is R2+R3. The circuit decreases to the one given in Figure 2.5(b). Only two resistances, R1 and (R2+R3), are connected in parallel in this circuit. Hence, R_{eq} can be determined as

$$R_{eq} = \left[\frac{1}{R1} + \frac{1}{R2+R3}\right]^{-1} = \frac{R1(R2+R3)}{R1+R2+R3}.$$

Another resistive circuit is depicted in Figure 2.6(a). In this case we must reduce the two parallel resistors across the terminals *b-c* into a single resistance first, as shown in Figure 2.6(b). R1 is now series with the parallel combination of R2 and R3. R_{eq} can now be found readily as

$$R_{eq} = R1 + \frac{R2\,R3}{R2+R3},$$

as shown in Figure 2.6(c).

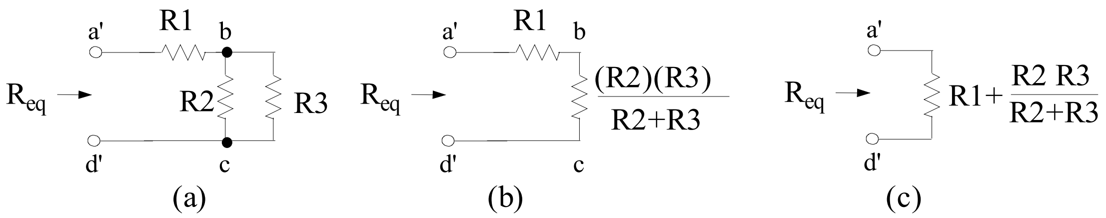


Figure 2.6 A resistive circuit

2.3. Analysis of electrical circuits

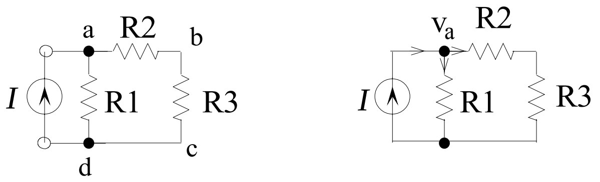
The knowledge of the value of current through each branch or the voltage across each element is often required. The circuits are analyzed to find these quantities. There are two methods of analysis. The first one is the *node-voltage* method, or *node analysis*.

A node is a point in the circuit where more than two elements are connected together. For example *b* in Figure 2.6(a) is a node but *b* in Figure 2.5(a) is not.

We follow a procedure outlined below to carry out the node analysis:

1. Select a common node so that all other node voltages are defined with respect to this node. Usually zero-potential node or *ground* node is taken as common node in electrical circuits.
2. Define the voltage difference between the common and all other nodes as unknown *node voltages*.
3. Write down the KCL at each node, expressing the branch currents in terms of node voltages and sources.
4. Solve the equations obtained in step 3 simultaneously.
5. Find all branch currents and voltages in terms of node voltages.

Consider the circuit in Figure 2.5(a), with a current source connected across terminals *a'-d'*. This circuit is given in Figure 2.7(a) with numerical values assigned to circuit parameters.



(a) (b)

$I = 1\text{A}$
R1=10 Ω
R2=100 Ω
R3=100 Ω

Figure 2.7 Node analysis

Let us analyze this circuit to find all element voltages and currents using node analysis procedure:

1. Choice of common node is arbitrary; we can choose either *a* or *d*. Choose *d*.
2. Define the node voltage v_a as the potential at node *a* minus the potential at node *d*, as shown in Figure 2.7(b).
3. KCL at node *a* (considering the assumed directions of flow):
   Source current (flowing into the node) = Current through R1(flowing out) + Current through R2 and R3 (flowing out)
   or
   $$I = \frac{v_a}{R1} + \frac{v_a}{R2 + R3}.$$
4. Solve for v_a in terms of the source current and resistors:

$$v_a = \frac{I}{\dfrac{1}{R1} + \dfrac{1}{R2 + R3}} = I\,\frac{R1(R2 + R3)}{R1 + R2 + R3}$$

$$= 9.52 \text{ V}.$$

5. Branch currents in terms of v_a are already given in step 3. Current flowing through R1 is v_a/R1, or 0.952 A, flowing from node *a* to node *d*. Current flowing through R2 and R3 is v_a/(R2+R3), or 47.6 mA.

   The voltage across R3 is (R3) [v_a/(R2+R3)] = (100 Ω)(47.6 mA) = 4.76 V. Similarly, voltage across R2 can be found as 4.76 V, either from (R2) [v_a/(R2+R3)] or from v_a− (R3) [v_a/(R2+R3)].

Hence all currents and voltages in the circuit are determined.
Another example is the circuit in Figure 2.6(a), driven by a voltage source as shown in Figure 2.8(a).

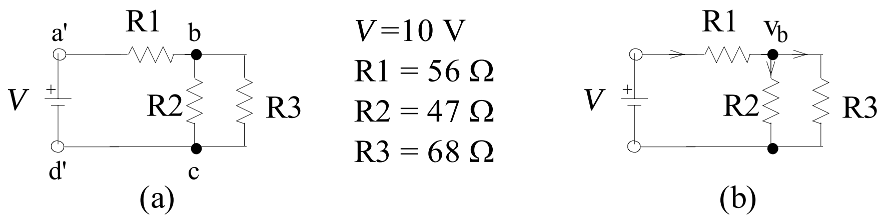


Figure 2.8 Node analysis

Node analysis procedure for this circuit is as follows:

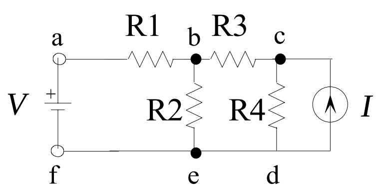
1. Chose node *c* as common.
2. Define the node voltage v_b.
3. KCL at node *b*:
   Current through R2 (out of node)+ Current through R3 (out of node) = Current through R1 (from the supply to the node)
   Or
   $$\frac{v_b}{R2}+\frac{v_b}{R3}=\frac{V-v_b}{R1}$$
4. Solve for v_b:
   $v_b = V\,(R1||\ R2||R3)/R1$
   $= 3.3$ V.
5. Voltage across R1 is $V - v_b = 6.7$ V. Current through R2 and R3 are v_b/R2 = 70.6 mA and v_b/R3 = 48.8 mA, respectively.

There is only one node voltage and therefore only one equation is obtained in step 3, in both of above examples. Consider the circuit in Figure 2.9(a).

Node analysis for this circuit is as follows:

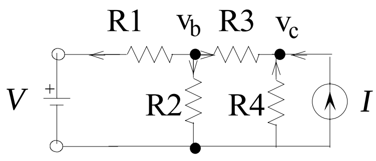
1. Connection points *d*, *e* and *f* are the same node in this circuit. Chose this point as common node.
2. Two other nodes are *b* and *c*. Define v_b and v_c.
3. KCL at node b:
   Choice of current directions is arbitrary (if the initial choice is opposite to that of actual flow for a branch, the solution comes out with negative sign). Let us

choose current directions such that all branch currents flow out of node b and all branch currents flow into node c, as shown in Figure 2.9(b).



(a)

V=10 V
I= 1 A
R1 = 56 Ω
R2 = 47 Ω
R3 = 68 Ω
R4 = 22 Ω



(b)

Figure 2.9 Two-node circuit

All currents coming out of node *b* = 0, or

$$\frac{v_b - V}{R1} + \frac{v_b}{R2} + \frac{v_b - v_c}{R3} = 0 \Rightarrow v_b\left(\frac{1}{R1} + \frac{1}{R2} + \frac{1}{R3}\right) - \frac{v_c}{R3} = \frac{V}{R1}.$$

KCL at node c:
All currents flowing into node *c* = 0, or

$$\frac{v_b - v_c}{R3} + \frac{-v_c}{R4} + I = 0 \Rightarrow \frac{v_b}{R3} - v_c\left(\frac{1}{R3} + \frac{1}{R4}\right) = -I.$$

4. KCL yields two equations for node voltages v_b and v_c, in terms of known resistances and source values. Solving them simultaneously yields v_b = 8.4 V and v_c = 18.7 V.
5. R1 current (in the chosen direction) is $(v_b - V)/R1$ = – 28.2 mA;
   R2 current (in the chosen direction) is $v_b/R2$ = 179 mA;
   R3 current (in the chosen direction) is $(v_b - v_c)/R3$ = – 151 mA;
   R4 current (in the chosen direction) is $-v_c/R4$= – 849 mA.
   The branch currents with negative values flow in the direction opposite to the one chosen initially.

The other method of circuit analysis is called the *mesh* analysis. In this method, the currents around the loops in the circuit are defined as mesh currents, first. Then KVL is written down for each mesh in terms of mesh currents. The two methods are mathematically equivalent to each other. Both of them yield the same result. Mesh analysis is more suitable for circuits, which contain many series connections. Node analysis, on the other hand, yields algebraically simpler equations in most electronic circuits.

2.4. Capacitors

Capacitors are charge storage devices. In this respect they resemble batteries. However capacitors store electrical charge again in electrical form, whereas batteries store electrical energy in some form of chemical composition. Charge, Q, stored in a capacitor is proportional to the voltage, V, applied across it:

$$C = \frac{Q}{V},$$

where C is the *capacitance* of the capacitor, and is measured in farads (F), and charge Q is in *coulombs*. Note that capacitance differs from a resistance, where current through a resistance is proportional to the voltage across it.

The relation between the charge on a capacitor and the current through it is somewhat different and *time dependent*. If we let a current, i(t), of arbitrary time waveform to pass through a capacitor, the amount of charge accumulated on the capacitor within a time interval, e.g. $(0,t_1)$, is given as

$$Q=\int_0^{t_1} i(t)dt.$$

If i(t) is a d.c. current, I_{dc}, then the charge accumulated on the capacitor is simply $Q=I_{dc}t_1$. For example a d.c. current of 1 mA will accumulate a charge of 10^{-9} coulomb on a capacitor of 1 μF in 1 μ-second. This charge will generate 1mV across the capacitor.

More generally this relation is expressed as

$$Q(t)=\int_0^{t} i(\xi)d\xi,$$

where the difference between the real (may be *present* or *observation*) time instant **t** and the integration time variable ξ, which operates on all passed time instances between 0 and **t**, is emphasized. Using C = Q/V, we can now relate the voltage across a capacitor and the current through it:

$$v(t)=\frac{1}{C}\int_0^{t} i(\xi)d\xi.$$

If we differentiate both sides of this equation with respect to t, we obtain

$$i(t)=C\frac{d}{dt}v(t).$$

Hence the current through a capacitor is proportional to the time derivative of voltage applied across it.

There are two major types of capacitors. The first type is *non-polar*, i.e. the voltage can both be positive and negative. Most of the capacitors of lower capacitance value are of this type. However as the capacitance values become large, it is less costly to use capacitors, which has polarity preferences, like *electrolytic* or *tantalum* capacitors. For these capacitors the voltage must always remain positive in the sense indicated. Symbols of both types are depicted in Figure 2.10(a).

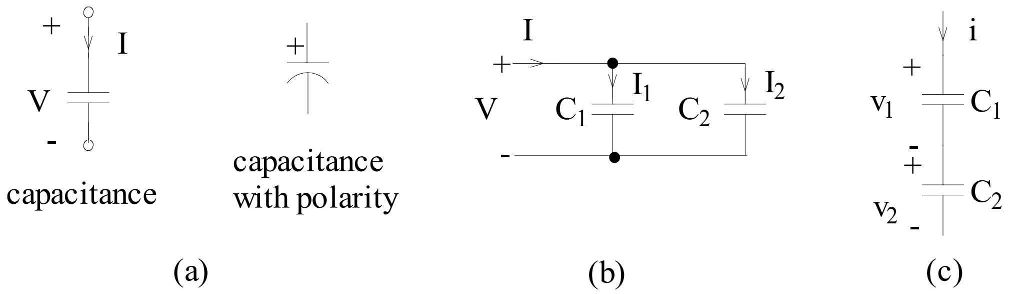


Figure 2.10 Capacitor circuits. (a) capacitance, (b) parallel connected capacitances, and (c) series connected capacitances

When capacitors are connected in parallel, as shown in Figure 2.10(b), I_1 will charge C_1 and I_2 will charge C_2. If Q_1 and Q_2 are the charges accumulated on these two capacitors respectively, then the total charge is,

$Q=Q_1+Q_2= C_1V+ C_2V=(C_1+ C_2)V$.

Hence when capacitors are connected in parallel, total capacitance is $C= C_1+ C_2$.

KVL tells us that in any loop, the sum of branch voltages is zero. When the capacitors are connected in series, as shown in Figure 2.10(c), sum of branch voltages is $v_1+ v_2 = v$ and hence,

$$v = v_1 + v_2 = \frac{1}{C_1}\int_0^t i(\xi)d\xi + \frac{1}{C_2}\int_0^t i(\xi)d\xi = \left[\frac{1}{C_1} + \frac{1}{C_2}\right]\int_0^t i(\xi)d\xi$$

Therefore, when the capacitors are connected in series, total capacitance is

$$C = \left[\frac{1}{C_1} + \frac{1}{C_2}\right]^{-1}$$

2.4.1. Power and energy in capacitors

The power delivered to the capacitor is

$$P(t) = v(t)i(t) = C\, v(t)\frac{dv(t)}{dt}.$$

P(t) can be written as

$$P(t) = \frac{d}{dt}\left(C\frac{v^2}{2}\right)$$

which cannot contain a non-zero average term. Therefore, the average power delivered to a capacitor is always zero, whatever the voltage and current waveforms may be. Physically this means that capacitors cannot dissipate energy.

In case of sinusoidal voltages, with

$v(t) = V_1\sin(\omega t+\theta)$, and

$$i(t) = C\frac{dv(t)}{dt} = \omega CV_1\cos(\omega t+\theta),$$

P(t) becomes

$P(t)= (\omega CV_1^2/2)\sin(2\omega t+2\theta)$.

On the other hand, capacitors store energy. If we sum up (or integrate) the power delivered to the capacitor we must obtain the total energy delivered to it. Assuming that initially capacitor has no charge, i.e. $v_C(0) = 0$, and v(t) is applied across the capacitor at t =0, the energy delivered to the capacitor at time t_1 is,

$$E = \int_0^{t_1} P(t)dt = \int_0^{t_1} d\{C\frac{v^2(t)}{2}\} = C\frac{v^2(t_1)}{2}$$

If the applied voltage across the capacitor is a d.c. voltage V_{dc}, the energy stored in the capacitor is

$$E = C\frac{V_{dc}^2}{2}.$$

2.4.2. RC circuits

We combine resistors and capacitors in electronic circuits. When a resistor is connected to a charged capacitor in parallel, as in Figure 2.11, the circuit voltages become a function of time. Assume that initially capacitor is charged to V_o volts (which means that it has $Q = CV_o$ coulombs stored charge). At t =0 we connect the resistance, R. KCL says that

$$i = C\frac{d}{dt}v_C(t) = C\frac{d}{dt}v(t)$$

and

$$i = \frac{v_R}{R} = -\frac{v}{R}$$

since $v_C = v$ and $v_R = -v$ at all times. In other words

$$\frac{dv}{dt} + \frac{v}{RC} = 0.$$

This equation is called a *first order differential equation.* Its solution for t ≥0 is

$v(t) = V_o\exp(-t/RC)$ for $t \geq 0$.

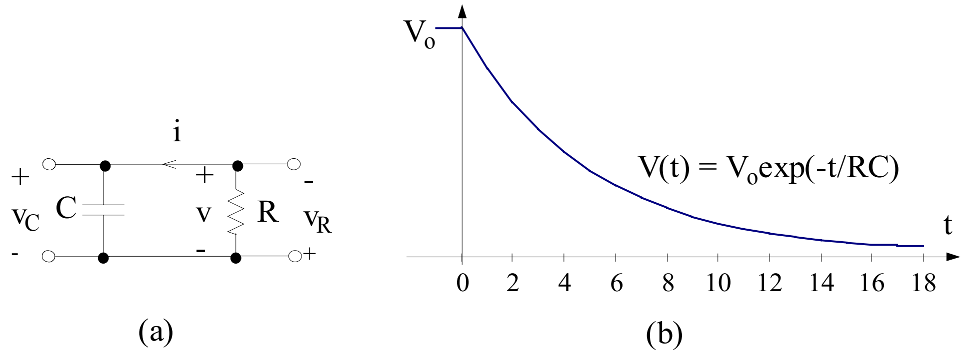


Figure 2.11 RC circuit

v(t) is shown in Figure 2.11(b). Above expression tells that as soon as the resistor is connected the capacitor voltage starts decreasing, i.e. it discharges on R. The speed with which discharge occurs is determined by $\tau = RC$. $\tau = RC$ is called *time constant* and has units of time ($1\Omega X 1F=1$ second).

The current, on the other hand, is

$$i(t) = C\frac{dv}{dt} = -\frac{V_o}{R}\exp(-t/RC) \qquad \text{for } t \geq 0$$

Also $v_C(t)= -v_R(t)= v(t)$.

We found the above differential equation by using KCL. We could use KVL, in which case sum of the voltages, v_C and v_R ($=iR$), in the loop must yield zero:

$$v_C + v_R = v_C + iR = v + RC\frac{dv}{dt} = 0,$$

which is the same differential equation. Note that we used a *sign convention* here. *Once we have chosen the direction of current, the sign of the voltage on any element must be chosen such that the positive terminal is the one where the current enters the element.* Hence in this circuit the current is chosen in the direction from R to C at top, and thus the polarity of capacitor voltage v_C is similar to v, whereas the polarity of the resistor voltage v_R must be up side down. On the other hand, current actually flows from C to R on top of the figure, in order to discharge C. Therefore the value of the current is found negative, indicating that it flows in the direction opposite to the one chosen at the beginning.

Usually we do not know the actual current directions and voltage polarities when we start the analysis of a circuit. We assign directions and/or polarities arbitrarily and start the analysis. But we must carefully stick to the above convention when writing down KVL and KCL equations, in order to find both the values and the signs correctly, at the end of the analysis. The importance of this matter cannot be over emphasized in circuit analysis, indeed in engineering.

The *magnitude* of the current in the above circuit is at its maximum, V/R, initially, and decreases towards zero as time passes. This is expected, as the voltage across the capacitor similarly decreases.

If we connect a voltage source, $v_S(t)$, in parallel to the circuit in Figure 2.11, then the voltage on the capacitance is $v_S(t)$ directly. This means that since voltage source can supply indefinite amount of current when demanded, capacitor can charge up to the value of the voltage source at any instant of time without any delay. Similarly the current through the resistor is simply $v_S(t)/R$. This is shown in Figure 2.12(a).

Here $v_C(t)= -v_R(t)= v_S(t)$ at all times, and $i = - v_S(t)/R$.

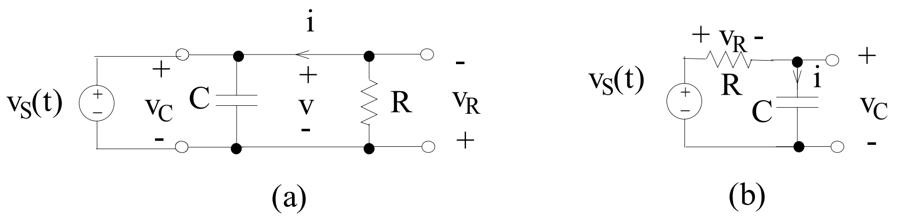


Figure 2.12 Parallel and series RC circuit

Figure 2.12(b) shows the case when R is connected in *series* to the capacitor instead of *parallel* in the same circuit. Assumed polarities (of voltage) and directions (of current) are clearly shown. The simplest way of analyzing this circuit is to apply KVL:

$v_C(t)+ v_R(t)- v_S(t)=0.$

The terminal relations of the two components are:

$v_R(t)= R\, i(t)$

and

$$i(t) = C\frac{dv_C(t)}{dt},$$

with given polarities and directions. Substituting the terminal relations into KVL equation, we obtain,

$$v_C(t) + RC\frac{dv_C(t)}{dt} = v_S(t).$$

The solution of this equation for a general time function $v_S(t)$ requires the knowledge of differential equations. However, we do not need this knowledge for the scope of this book. We shall confine our attention to the types of functions that represents the signals we shall use in TRC-10. Let us assume that $v_S(t)$ is zero until t= 0 and is a constant level V_S afterwards. Such time waveforms are called *step functions*. This is similar to a case of replacing $v_S(t)$ with a battery and a switch connected in series with it, and the switch is closed at t=0. This is modeled in Figure 2.13(a).

In order to find how $v_C(t)$ behaves for all t>0, we must know its value just before the switch is closed. We call this value, the *initial value*, $v_C(0)$. Let us also assume that $v_C(0) = 0$. We can see that $i(t) = 0$ and $v_C(t) = v_R(t) = 0$ for t<0 by inspecting Figure 2.12(b). When the supply voltage jumps up to V_S at t = 0, current i(t) starts flowing from the supply to the capacitor through R. Hence the circuit current magnitude cannot be any larger than V_S /R. R limits the amount of current that capacitor can drain from the supply, because it is connected in series. Hence the voltage across the capacitor, in this case cannot follow the changes in the supply voltage immediately.

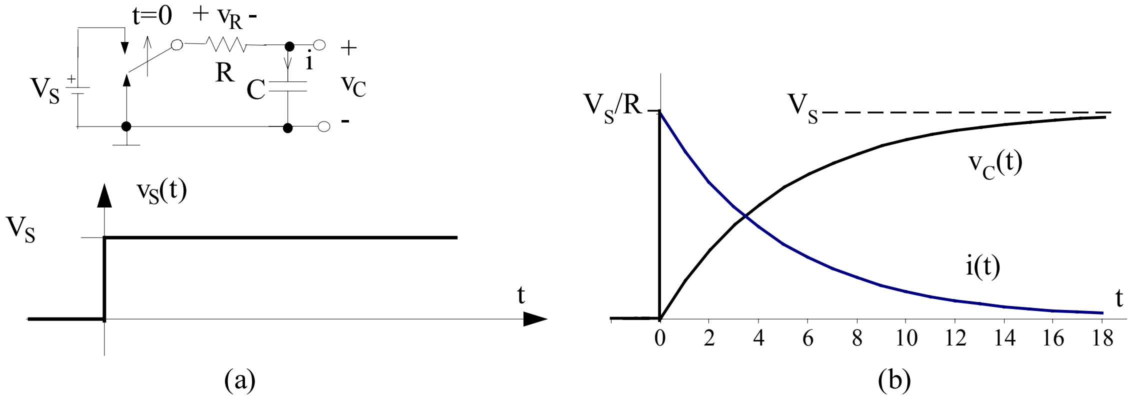


Figure 2.13 (a) $v_S(t)$ as step voltage, (b) i(t) and $v_C(t)$ vs. time

Initially, just after t = 0, the amount of current in the circuit is $i(0) = [V_S - v_C(0)]/R = V_S/R$. With this initial current capacitor starts charging up until the amount of charge accumulated on it reach to $Q = CV_S$. When this happens, the voltage across the capacitance is equal to that of the supply and it cannot charge any more. Hence there must not be any current flowing through it, which means that i(t) must become zero eventually.

As a matter of fact, if we write the KVL equation

$$v_C(t) + RC\frac{dv_C(t)}{dt} = V_S$$

for t>0, in terms of i(t) instead of $v_C(t)$ (i.e. differentiating the entire equation first and then substituting i(t)/C for $dv_C(t)/dt$), we have

$$i(t) + RC\frac{di(t)}{dt} = 0.$$

In order to obtain this, we first differentiate $v_C(t)$ equation and then substitute i(t)/C for dv_C/dt. This equation is similar to the case of parallel RC, and its solution is

$$i(t) = \begin{cases} 0 & \text{for } t<0 \\ (V_S/R)\exp(-t/RC) & \text{for } t \geq 0 \end{cases}$$

since the initial value of i(t) is V_S/R, as we determined by inspection above. Note that i(t) becomes zero as t becomes indefinitely large. Having found i(t), we can write down the circuit voltages by direct substitution:

$$v_R(t)=R\,i(t)=\begin{cases}0 & \text{for } t<0\\ V_S\exp(-t/RC) & \text{for } t\geq 0\end{cases}$$

and

$$v_C(t)=\frac{1}{C}\int_0^t i(\xi)d\xi$$
$$=V_S[1-\exp(-t/RC)] \qquad \text{for } t\geq 0$$

since we assumed $V_C(0)=0$ above. i(t) and $v_C(t)$ are depicted in Figure 2.13(b).

2.5. Diodes

Diodes are *semiconductor* devices. They are nonlinear resistors, resistance of which depends on the voltage across them. We use different types of diodes in TRC-10 circuit:

1N4001	silicon power diode
1N4448	silicon signal diode
MPN3404 (or BA479)	PIN diode
KV1360NT	variable capacitance diode (varicap)
	zener diode

and a diode bridge rectifier. The I-V characteristics of a diode can be well approximated by an exponential relation:

$$I_D=I_S(\exp(V_D/\gamma)-1)$$

where I_D and V_D are current and voltage of the diode and I_S and γ are the physical constants related to the material and construction of the diode. The symbol for a diode and typical I-V characteristics of a *silicon* diode are given in Figure 2.14.

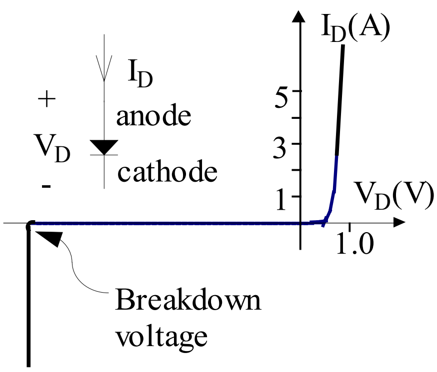


Figure 2.14 Diode

V_D is defined as the voltage difference between *anode* and *cathode* terminals of the diode. This I-V characteristics shows that the current through a diode is effectively zero as long as the voltage across it is less than approximately 0.7 volt, i.e. it can be assumed open circuit. The current increases very quickly when the voltage exceeds 0.7 volt, hence it behaves like a short circuit for these larger voltages. Also note here that there is a negative voltage threshold for V_D, determined by the *breakdown voltage* in real diodes, below which the diode starts conducting again. This breakdown voltage is usually large enough such that the magnitudes of all prevailing voltages in the circuit are below it, and hence it can be ignored. In *zener diodes*, however, this breakdown voltage is employed to stabilize d.c. voltage levels.

It is useful to introduce the concept of *ideal diode* at this stage in order to model real diodes in circuit theory. Ideal diode is a device, which is an open circuit for all negative voltages and is a short circuit for positive voltages. The symbol and I-V characteristic of an ideal diode is shown in Figure 2.15.

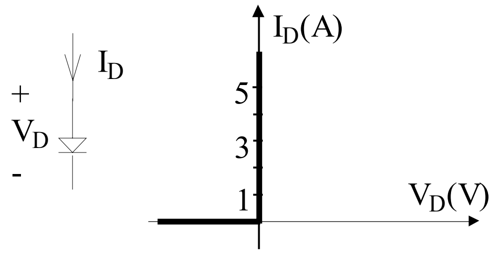


Figure 2.15 Ideal diode equivalent circuit and characteristics

Having defined the ideal diode, we can now develop approximate *equivalent circuits* for diodes. The simplest one is an ideal diode and a voltage source connected in series. In this model the voltage source represents the *threshold voltage* V_o (≈0.7 V), we observed previously. This equivalent circuit and its I-V characteristics are shown in Figure 2.16(a).

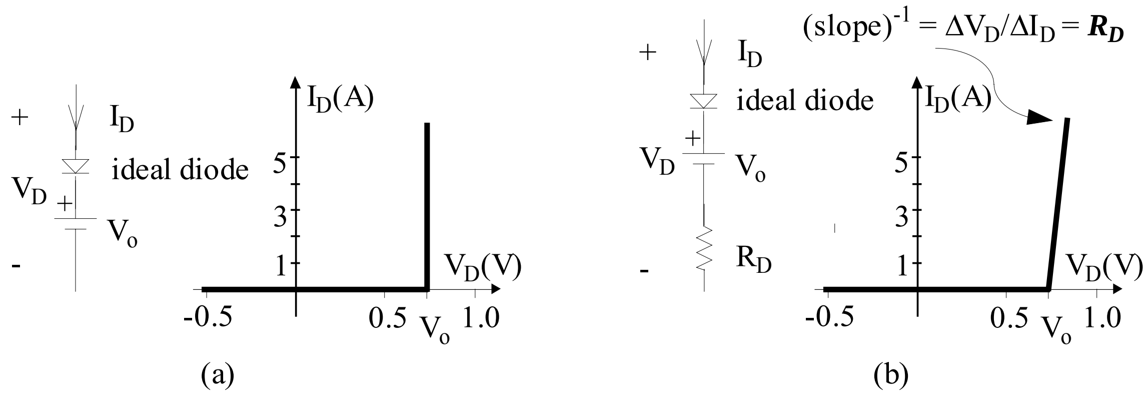


Figure 2.16 Piecewise linear diode equivalent circuits and characteristics

The equivalent circuit given in Figure 2.16(a) is a good enough approximation for many applications. However when the *on resistance* of the diode, R_D (i.e. the incremental resistance when V_D is larger than V_o), becomes significant in a circuit, we can include R_D to the equivalent circuit as a series resistance as shown in Figure

2.16(b). A diode is called "*ON*" when it is conducting, and "*OFF*" otherwise. Note that R_D is zero in the simpler model.

2.5.1. Diodes as rectifiers

Diodes are used for many different purposes in electronic circuits. One major application is *rectification*. Electrical energy is distributed in form of alternating current. Although this form of energy is suitable for most electrical appliances, like machinery, heating and lighting, direct current supplies are necessary in electronic instrumentation. Almost all electronic instruments have a *power supply* sub-system, where a.c. energy supply is converted into d.c. voltage supplies in order to provide the necessary energy for the electronic circuits. We first rectify the a.c. voltage to this end. Consider the circuit depicted in Figure 2.17 below.

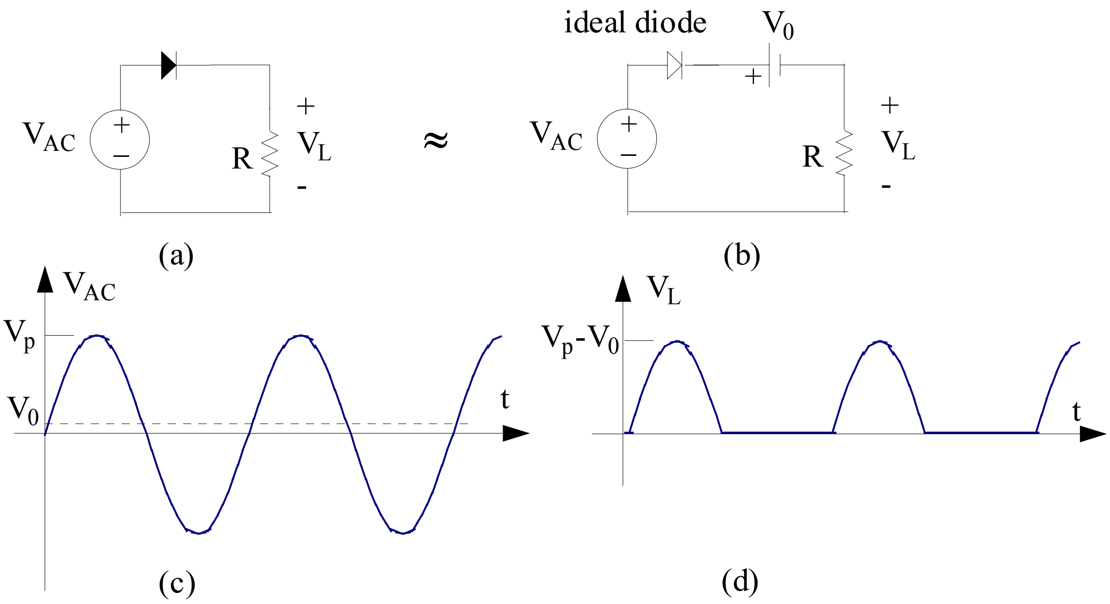


Figure 2.17 (a) Diode rectifier, (b) equivalent circuit, (c) input a.c. voltage and (d) voltage waveform on load resistance.

There is a current flowing through the circuit in Figure 2.17(b) (or (a)) during the positive half cycles of the a.c. voltage v_C at the input, while it becomes zero during negative half cycles. Here we assume that V_p is less than the breakdown voltage of the diode. The current starts flowing as soon as v_{AC} exceeds V_o and stops when v_{AC} falls below V_o. The voltage that appears across the load, v_L, is therefore sine wave tips as depicted in Figure 2.17(d). This voltage waveform is neither an a.c. voltage nor a d.c. voltage, but it is always positive.

When this circuit is modified by adding a capacitor in parallel to R, we obtain the circuit in Figure 2.18(a), and its equivalent Figure 2.18(b). The capacitor functions like a filter together with the resistance, to smooth out v_L.

When v_{AC} in Figure 2.18(c) first exceeds V_o (at $t = t_1$), diode starts conducting and the current through the diode charges up the capacitor. The resistance along the path is the diode *on* resistance. In the equivalent circuit we have chosen for this application (Figure 2.18(b)) this resistance is zero so that the time constant for charging up period is also zero. This means that the capacitance voltage, hence v_L, will follow v_{AC} instantly. Charge up continues until v_L reaches the peak, V_p-V_o, at $t = t_2$. After peak voltage is reached, the voltage at the anode of the diode, v_{AC}, falls below V_p and hence the voltage across the diode, $V_D= v_{AC} - v_L$, becomes negative. The current through the diode cease flowing. We call this situation as the *diode is reverse biased*. Now we have a situation where the a.c. voltage source is isolated from the parallel RC circuit, and the capacitor is charged up to V_p-V_o. Capacitor starts discharging on R with a time constant of RC. If RC is small, capacitor discharges quickly, if it is large, discharge is slow. The case depicted in Figure 2.18 (c) is when RC is comparable to the period of the sine wave.

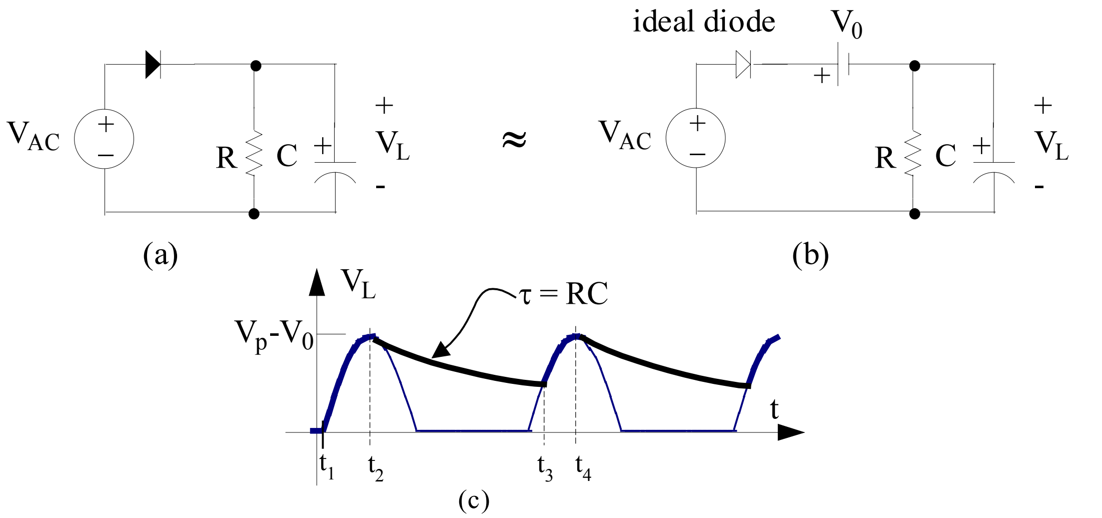


Figure 2.18 (a) Diode rectifier with RC filter, (b) equivalent circuit and (c) voltage waveform on load resistance.

As v_{AC} increases for the next half cycle of positive sine wave tip, it exceeds the voltage level to which the capacitor discharged until then, at $t = t_3$, and *diode is switched on* again. It starts conducting and the capacitor is charged up to V_p- V_o all over again ($t = t_4$).

The waveform obtained in Figure 2.18(c) is highly irregular, but it is obviously a better approximation to a d.c. voltage compared to the one in Figure 2.17(d).

We preferred electrolytic capacitor in this circuit, which possesses polarity. In this circuit, the voltages that may appear across the capacitor is always positive because of rectification, and hence there is no risk in using such a capacitor type. On the other hand, large capacitance values can be obtained in small sizes in these type of capacitors. Large capacitance values allow us to have more charge storage for the same voltage level, large time constant even with smaller parallel resistances, and thus smoother output waveforms.

A better way of rectifying a.c. voltage is to use four diodes instead of one, as shown in Figure 2.19. In this case we utilize the negative half cycles as well as positive ones.

The four-diode configuration is called a *bridge*, and the circuit is called *bridge rectifier*.

When v_{AC} is in its positive phase, D2 and D4 conducts, and current flows through D2, capacitor and D4, until capacitor is charged up to the peak value, V_p-2V_0. The peak voltage for v_L is less than the one in single diode case, because the charging voltage has to overcome the threshold voltage of two diodes instead of one. During the negative half cycles, D1 and D3 conducts and the capacitor is thus charged up in negative phase as well. Since the capacitor is charged twice in one cycle of v_{AC} now, it is not allowed to discharge much. The waveform in Figure 2.19(c) is significantly improved towards a d.c voltage, compared to single diode case.

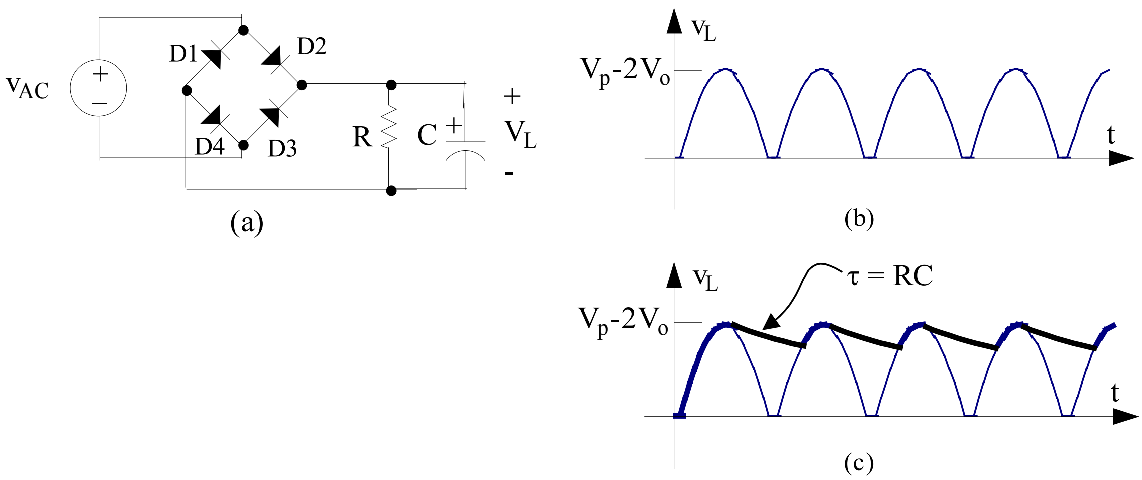


Figure 2.19 (a) Bridge rectifier, (b) rectified voltage without capacitor, and (c) filtered output voltage

2.5.2. Zener diodes as voltage sources

Zener diodes are used as d.c. voltage reference in electronic circuits. Zener diodes are used in the vicinity of breakdown voltage as shown in Figure 2.20, as opposed to rectification diodes. The symbol for zener diode is shown in the figure.

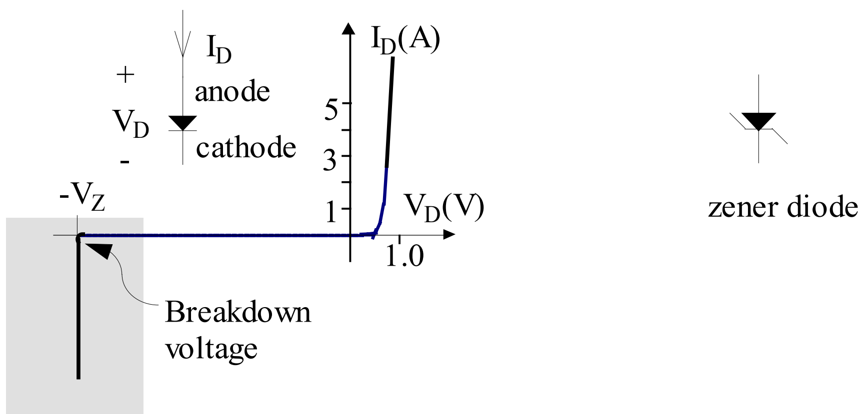


Figure 2.20 Zener diode and its characteristics

When a zener diode is used in a circuit given in Figure 2.21(a), a reverse diode current

$-I_D = (V_S\text{-}V_Z)/R$

flow through the diode as long as $V_S > V_Z$. V_Z appears across the diode. When V_S is less than V_Z, the diode is no longer in the breakdown region and it behaves like an open circuit.

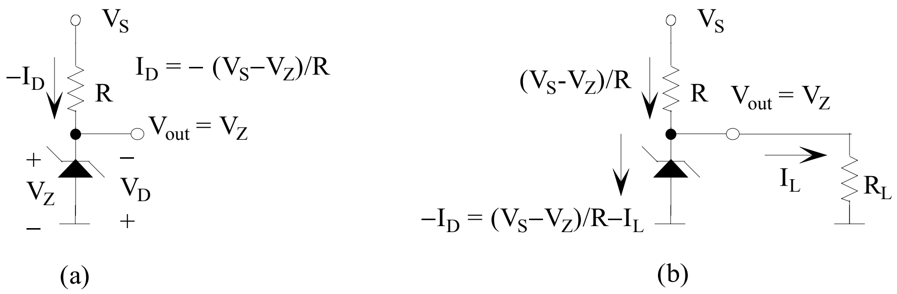


Figure 2.21 Zener diode in a voltage reference circuit

Assume that a load resistance R_L is connected across the diode, as shown in Figure 2.21(b). The current through R_L is

$I_L = V_Z/R_L$.

As long as the diode current $-I_D$ is larger than zero, i.e.

$I_L < (V_S\text{-}V_Z)/R$,

diode remains in breakdown region and provides the fixed voltage V_Z across its terminals.

2.6. Inductors

When current flow through a piece of wire, a magnetic flux is generated around the wire. Reciprocally, if a conductor is placed in a time varying magnetic field, a current will be generated on it. From the electrical circuits point of view, this phenomena introduces the circuit element, *inductor*. Inductors are flux or magnetic energy storage elements. Inductance is measured in *Henries* (*H*), and since most inductors used in electrical circuits have the physical form of a wound coil, a coil symbol is used in circuit diagrams to represent inductance (Figure 2.22(a)). The terminal relations of an inductance is given as

$$v(t) = L\frac{di(t)}{dt}$$

where v(t) and i(t) are current through and voltage across the inductance, and *L* is the value of inductance in henries. We note that voltage is proportional to the time derivative of current in an inductor. If i(t) is a d.c. current, its derivative is zero, and hence the voltage induced across inductor is zero. Putting this in another way, if we apply a d.c. voltage across an inductor, the current that will flow through the inductor will be indefinitely large (= ∞), and the applied voltage is shorted.

TRC-10 employs few different types of inductors. Some inductors are made by simply shaping a piece of wire in the form of a helix. These are called *air core inductors*. When larger inductance values are required in reasonable physical sizes, we wind the wire around a material which has higher *permeability* compared to air. This material is referred to as *core*, and such inductors are symbolized by a bar next to the inductance symbol, as shown in Figure 2.22(a).

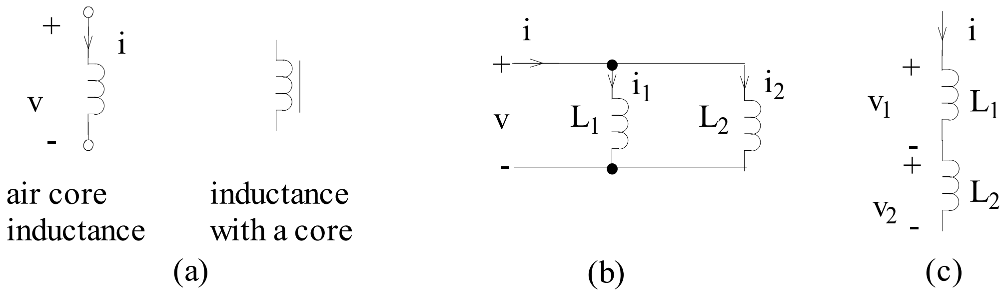


Figure 2.22 Inductor circuits. (a) inductance, (b) parallel inductances, and (c) series inductances.

The parallel and series combination of inductors are similar to resistance combinations, as can be understood from the terminal relation above. For parallel connected inductors as in Figure 2.22(a), the total inductance is

$$L = \left(\frac{1}{L1} + \frac{1}{L2}\right)^{-1},$$

whereas for series connected inductors (Figure 2.22(c)),

$$L = L_1 + L_2 .$$

The net magnetic energy stored in an inductor in the interval $(0,t_1)$ is

$$E = \int_0^{t_1} P(t)dt = \int_0^{t_1} v(t)i(t)dt = \int_0^{t_1} \left(L\frac{di(t)}{dt}\right)i(t)dt = L\int_0^{t_1} d\left(\frac{i^2(t)}{2}\right) = L\frac{i^2(t_1)}{2} - L\frac{i^2(0)}{2}$$

If i(t) above is a d.c. current applied at t = 0, i.e. $i(t) = I_{dc}$ for $t \geq 0$ and $i(t) = 0$ for $t < 0$, then the net energy stored in the inductor is

$$E = L\frac{I_{dc}^2}{2}.$$

When we connect a parallel resistance to an inductor with such stored energy, $E = LI_i^2/2$ (we changed I_{dc} to I_i to avoid confusion), we have the circuit shown in Figure 2.23(a). Initially the inductor current i_L is equal to $i(0) = I_i$. Since R and L are connected in parallel, they have the same terminal voltage:

$$v = Ri_R = L\frac{di_L}{dt}$$

and

$i(t) = i_L = -\, i_R.$

Figure 2.23 LR circuit

Therefore

$$\frac{di(t)}{dt} + \frac{R}{L} i(t) = 0\,.$$

This equation is again a differential equation similar to capacitor discharge equation. Its solution is also similar:

$i(t) = I_i \exp(-t/\tau)$ for $t \geq 0$.

where the time constant is $\tau = L/R$ in this case. Now let us assume that we connect a step voltage source $v_S(t)$, like the one in Figure 2.13(a), in series with R, instead of R alone. This circuit is given in Figure 2.23(b). Again the initial loop current is equal to the current stored in the inductor:

$i(0) = I_i$

and the KVL equation is

$$-v_S(t) + Ri(t) + L\frac{di(t)}{dt} = 0\,.$$

$v_S(t)$ is zero for $t<0$, and $v_S(t)= V_S$ for $t \geq 0$. If we differentiate the above equation with respect to t, we obtain

$$\frac{dv_S(t)}{dt} - R\frac{di(t)}{dt} - L\frac{d^2 i(t)}{dt^2} = 0\,.$$

$dv_S(t)/dt$ term is zero for $t > 0$, since $v_S(t)$ is constant. Substituting v_L/L for $di(t)/dt$ in the above equation, we obtain

$$\frac{dv_L(t)}{dt} + \frac{R}{L} v_L(t) = 0 .$$

We can write down the solution of this equation after determining the initial value of v_L. Just after t=0,

$v_L(0) = V_S - Ri(0) = V_S - R\,I_i$.

Hence

$v_L(t) = (V_S - R\,I_i)\exp(-t/\tau)$ for $t > 0$

where $\tau = L/R$, and

$$i(t) = \frac{1}{L}\int v_L(\xi)d\xi = (I_i - V_S/R)\exp(-t/\tau) + K_\infty$$

The integration constant K_∞ can be determined by using the initial value of i(t):

$i(0) = I_i = I_i - V_S/R + K_\infty \Rightarrow K_\infty = V_S/R$.

i(t) is drawn in Figure 2.24. V_S/R term is the value that the inductor current will reach at $t = \infty$. Indeed as time passes, the derivative term in KVL equation must diminish, since the variation with respect to time slows down. Then for large t, $i(t) \approx V_S/R$. This value that the solution of the differential equation reaches at $t = \infty$ is called *steady-state value*, in electrical engineering.

Once we determined the initial value and the steady-state value of the solution, we can write down the solution of a first order differential equation directly, as we have done above.

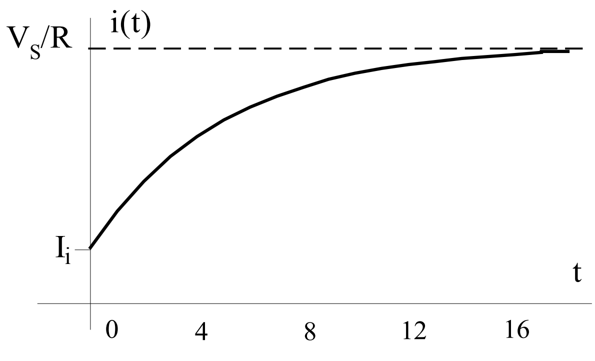


Figure 2.24 Inductor current in series RL circuit with step voltage input

2.7. Transformers

Transformers are two or more *coupled* inductors. They are coupled to each other by means of the same magnetic flux. In other words, they share the same flux. The circuit symbol of a transformer with two *windings* (i.e. two inductors) is given in Figure 2.25.

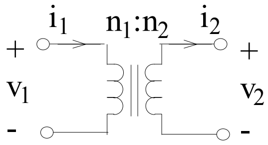


Figure 2.25 Transformer

The windings are referred to as *primary winding* and *secondary winding* respectively, in transformers. Transformers transform the voltage and current amplitude that appears across the primary winding to another pair of amplitudes at the secondary, and vice versa. The amount of transformation is determined by the *turns ratio* n_1:n_2. The relations in an *ideal transformer* are as follows:

$$\frac{v_2}{v_1} = \frac{n_2}{n_1}$$

where as

$$\frac{i_2}{i_1} = \frac{n_1}{n_2}.$$

There are four transformers in TRC-10. The first one is the *mains transformer* used in power supply sub-system, and other three are RF transformers. We shall delay a detailed discussion of transformers until we consider the RF circuits. The mains transformer, as used in TRC-10, can well be modeled as an ideal transformer.

2.8. Circuit Protection Devices

There is always a possibility that voltages much larger than envisaged levels can appear in electronic circuits. For example, when a lightening strikes to a power line, it is likely that very high voltage spikes can appear on the voltage supply. Similarly very high currents can be drawn from supplies because of mishandling, such as short circuits. It is likely that many of you will experience short circuits in the Lab Exercises of TRC-10. We use *varistors* (*Var*iable Res*istors*) as over voltage protection devices, and *positive temperature coefficient thermistors (PTC)* as over current protection devices.

2.8.1. Varistors

Varistors are nonlinear resistors made of ceramic-like materials like sintered zinc oxide or silicon carbide. The I-V characteristics of a varistor is depicted in Figure 2.26, together with its symbol.

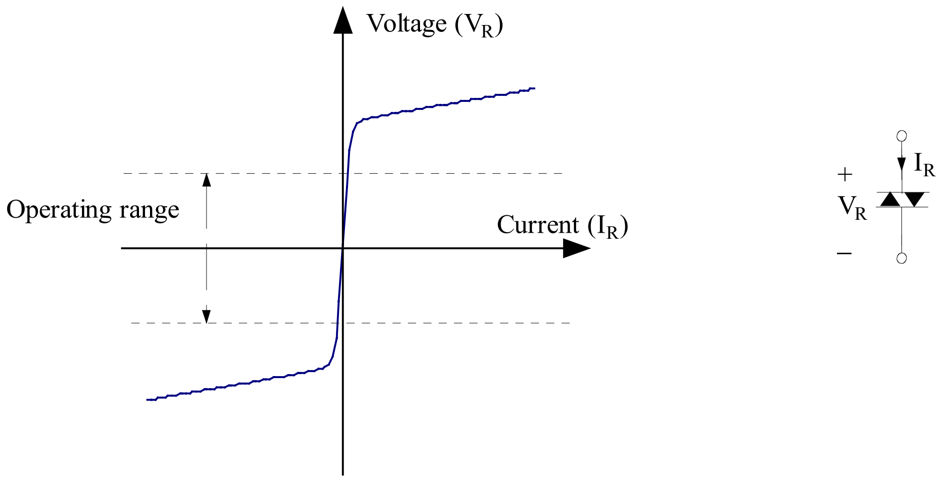


Figure 2.26 Varistor characteristics and symbol

When the voltage across the varistor is within the operating range, varistor exhibits a very large resistance. When the voltage increases, the resistance falls rapidly, thus taking most of the excess current due to over-voltage.

Varistors are connected in parallel to the circuits to be protected.

2.8.2. PTC Thermistors

A PTC thermistor is a thermally sensitive ceramic resistor. Its resistance increases abruptly with increasing temperature beyond a specified limit (reference temperature). Using PTC as resettable fuse relies on the following consideration:

The current through the PTC under normal operating conditions is sufficiently low. At this current level the power dissipated by the PTC *on* resistance is again low enough, such that the PTC temperature does not exceed the reference temperature. When a short circuit occurs, the current through the PTC increases. The power dissipation increases the temperature over the reference temperature and PTC *trips* to high impedance state.

PTC's are usually specified by two current parameters. *Rated current* (I_N) is the current level, below which the PTC reliably remains in low resistance mode. *Switching current* (I_S) is the level beyond which the PTC reliably trips to high resistance mode. Another parameter of significance is R_N, the resistance of PTC at low resistance mode.

PTC thermistors are connected in series to the circuit to be protected.

2.8.3. Circuit protection

An over-voltage protection circuit typically has the form shown in Figure 2.27.

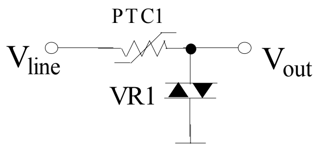


Figure 2.27 Over-voltage protection circuit

This circuit operates as follows:

VR1 and PTC1 are chosen such that, when there is not any over-voltage or surge current, the voltage across VR1 is in the normal range and the current through PTC1 is less than I_N. PTC exhibits a low resistance and VR exhibits a very high resistance.

When an over-voltage occurs on the line, the voltage across VR1 increases beyond its operating range. The current through VR1 increases rapidly due to the nonlinear nature of the varistor resistance. This current passes through PTC1 and warms the PTC up. PTC1 switches to high impedance mode isolating the line from the output, when this current exceeds I_S.

2.9. Bibliography

Chapter 24 in *ARRL Handbook* has comprehensive information and data on components.

There are excellent circuit theory books. *Electrical Engineering Uncovered* by D. White and R. Doering provides a very good introduction. N. Balabanian's *Electric Circuits* is one of excellent circuit theory books.

2.10. Laboratory Exercises

Power supply sub-system

1. The construction of TRC-10 starts with the power supply. Examine the circuit diagram of the power supply given in the appendix. Familiarize yourself with the components in this circuit.
2. Read all exercises in this chapter carefully.
3. Place the mounting tray on your desk.
4. Mount the mains jack J1.
5. Mount the fuse holder.
6. Mount the mains switch S1.
7. Mains switch is a *double pole single throw* (DPST) switch. This means that it has a pair of single switches, connects two pair of lines when closed (double pole), and only disconnects when open (single throw).We shall connect the mains live and neutral lines to the transformer primary winding through the mains switch. The mains switch is a toggle switch marked as "I/O". When set to "I" we want the mains connected, when set to "O", disconnected. A neon bulb is fitted internally between the two contacts on one side of the switches, as shown in the figure below. We want that side connected to our circuit, so that the neon will be energized only when TRC-10 is switched on. The other side is toggle side and must be connected to mains. Neon bulbs are gas discharge bulbs, and emit light

only when the voltage across them is larger than approximately 90 volts. They draw very little current when emitting light. They are commonly used power indicators. Check between which contacts the neon is fitted.

To make the *live* connection, cut two pieces of 10 cm long *brown* colored wire, strip both ends for about 5mm each, and tin them. Solder one end of the wires to one of the two circuit taps on the switch. Solder one end of the other piece of wire to the corresponding toggle tap on the switch. Cut two 2 cm long pieces of 6 mm diameter heat-shrink sleeve and work each wire through one of them. Push the sleeves as far as you can such that the sleeves cover the taps entirely.

Heat-shrink sleeves shrink to a diameter, which is 30% to 50% of its original diameter when exposed to heat of about 90^{o}C for a few seconds. It is an isolating material so that there will not be any exposed hot conducting surfaces. Take the hot air gun, adjust the temperature to 90^{o}C and shrink the sleeve. This can be done by a lighter or a hair dryer instead of hot air gun. Hot air gun is a professional tool and must be handled with care. It can blow out very hot air reaching to 400 ^{o}C and can cause severe burns. Ask the lab technician to check your work.

Cut two more pieces of heat shrink sleeve and a piece of 10 cm long brown wire. Tin the wire ends. Solder this wire to one tap of the fuse holder. Work the brown wire coming from the switch into a piece of sleeve and then solder it to the other tap of the fuse holder. Push the sleeve to cover the tap completely. Fit the remaining sleeve onto the other tap. Using hot air gun, shrink the sleeves.

Cut another piece of heat shrink sleeve and work the wire connected to the fuse holder through it. Solder the wire end to the hot tap on the mains jack. Push the sleeve so that it completely covers the tap. Using the hot air gun shrink the sleeve.

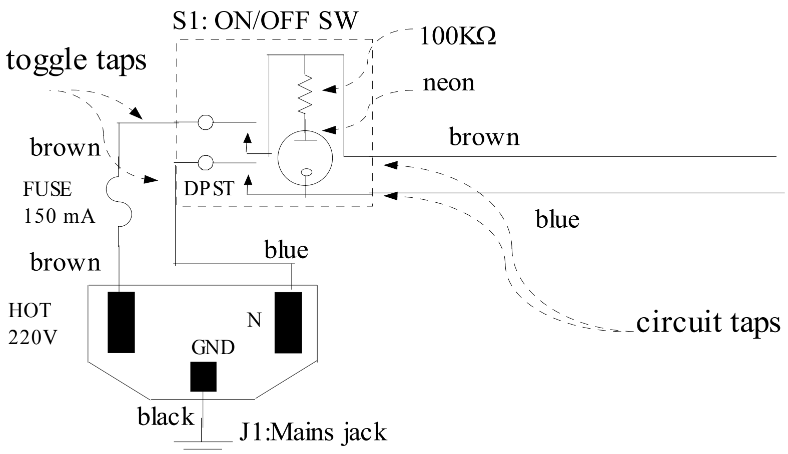


Figure 2.28

To make the *neutral* line connections, cut two 10 cm pieces of *blue* wire, and tin the ends. Make the connections to the other pair of taps on the switch, similar to the live one. Make sure that sleeves cover all conducting surfaces. ***Safety first!*** If you work carefully and tidily, there will never be any hazardous events. There must not be any hazardous events. Indeed a careful and clever engineer will **never** get a shock or cause any hazard to others.

Notice that we used a color code for mains connection: *brown* for live line and *blue* for neutral. We shall use *black* for earth connections.

8. Mount the mains transformer T1 on the TRC-10 tray, by means of four screws. Do not forget to use anti-slip washers. Otherwise the nuts and bolts may get loose in time.

9. Mains transformer is a 10 W 220V to 2X18 V transformer. That is, it converts 220V line to 18V a.c. There are two secondary windings, and hence we have two 18V outputs with a common terminal.

10. Cut two 2 cm pieces of heat shrink sleeve and work the two wires, brown and blue coming from the switch, into each one of the sleeves. Solder the wires to the two primary windings of the transformer. Push the sleeves so that they cover the transformer terminals completely. Shrink the sleeves using hot air gun.

Now we have completed the mains connections. *Root mean square* (rms) definitions of voltage and current prevail particularly for a.c. circuits. Line voltages have sinusoidal waveform. The frequency of the line, f_L, differs from country to country, but it is either 50Hz or 60Hz. For some specific environments there are other line frequency standards (in aircraft for example, a.c. power line is 400Hz). If we express the line voltage as

$$v(t)=V\sin(\omega t+\theta)$$

where $\omega=2\pi f_L$ and V is the amplitude of the line voltage, then *rms* value of V is defined as

$$V_{rms}=\sqrt{\frac{1}{T}\int_0^T v^2(t)dt}=\frac{V}{\sqrt{2}}$$

where T is the period o the sine wave. Similarly rms value of a sine wave current is

$$I_{rms}=\frac{I}{\sqrt{2}}$$

with I the current amplitude. Note that with this definition, on a line of V_{rms} potential and carrying I_{rms} current, the total power is $P=V_{rms}\, I_{rms}$.

When a line voltage is specified, e.g. *220 V*, it means that the line potential is 220 volts rms. Hence, the voltage on this line is of sinusoidal form with an amplitude of approximately (nominally) 310 V. Similarly, for a transformer specified as *220V to 18 V*, it means that the transformer transforms line voltage of 310 V amplitude to a sinusoidal voltage of 25.5 volts amplitude, at the line frequency.

As a matter of fact, if we measure an a.c. voltage with a multimeter, the reading will be the rms value of the voltage.

Power line voltages also differ from country to country, but there are only few standards. Line voltages are 110 V_{rms}, 120 V_{rms}, 220 V_{rms}, or 240 V_{rms}. The only component sensitive to the line voltage specification in TRC-10 is the line transformer. The power line in this environment is 50Hz/220 V_{rms} line and hence transformer is chosen accordingly.

Electric energy is generated in electric power plants. The generated power must be transported long distances before it can be used, since power plants can be quite far away to areas where large energy demand is. Voltage level is either 6.3 KV rms or 13.8 V rms at the terminals of the generator in the plant. In order to carry the power over long distances with minimum energy loss, the voltage of the line is stepped up to a very high level, usually to 154 or 380 KV_{rms}. The transport is always done by means of *high voltage* (HV) *overhead lines* (OHL). This voltage level is stepped down to a lower level of 34.5 KV *medium voltage* (MV), in the vicinity of the area (may be a town, etc.) where the energy is to be consumed. Energy is distributed at this potential level (may be up to few tens of km). It is further stepped down to household voltage level (e.g. 220 V- the voltage referred to as “220 V rms” actually means a voltage level between 220 to 230 V rms) in the close vicinity of the consumer. All this step-up and step-down is done by using power transformers.

We are accustomed to see the electric energy coming out of household system as a supply of single phase voltage on a pair of lines, *live* and *neutral*. When energy is generated at the generator, it always comes out in three phases. If the phase voltage that we observe between the live and neutral is

$v_1(t)=V_p\sin(\omega t)$,

then, it is always accompanied by two other related components,

$v_2(t)=V_p\sin(\omega t + 120^o)$ and

$v_3(t)=V_p\sin(\omega t - 120^o)$.

This is necessitated by the economics of the technology employed in electromechanical power conversion. These three phases of line supply is distributed to the consumers such that all three phases are evenly loaded, as much as possible.

As far as phase voltage is concerned, 220 V rms refers to the voltage difference between any one of the phase voltages and neutral. On the other hand, the potential difference between any two phases, which is called *line voltage*, e.g. between $v_1(t)$ and $v_2(t)$, is

$$\begin{aligned}\Delta v(t) &= v_1(t) - v_2(t)\\ &= \sqrt{3}V_p\sin(\omega t - 30^o)\end{aligned}$$

The potential difference between the phases is therefore 1.73 times larger that any one of phase voltage with respect to neutral. The line voltage level is 380-400 Vrms for a phase voltage of 220 V. The last step-down from MV to low voltage (LV) is depicted in Figure 2.29 below.

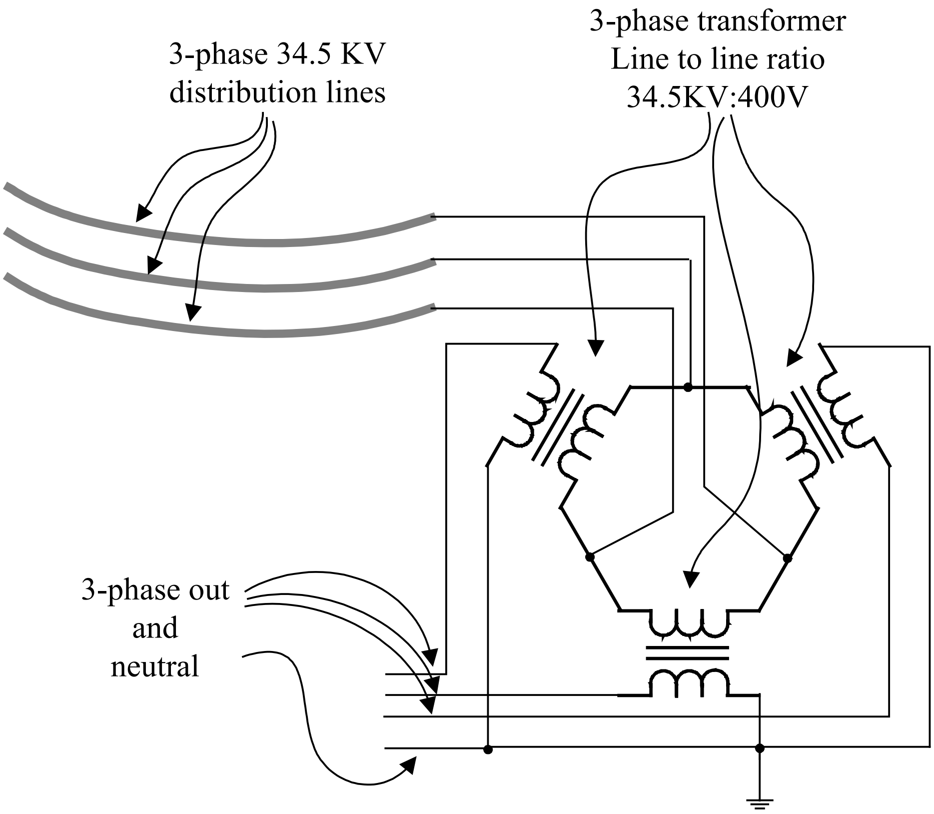


Figure 2.29 MV to LV transformer

Note that there is no neutral for 3-phase MV distribution lines (both HV and MV energy are carried as three phases only without neutral reference during the transportation). Once it is stepped down, one terminal of each of the secondary windings are grounded at the transformer site, and that node is distributed as neutral. Grounding is done by connecting that terminal to a large conducting plate or long conducting rods buried in earth. A separate line connected to earth is also distributed, since most household and professional equipment require a separate earth connection, not only for operational reasons but also for safety. Neutral is the return path of the current we draw from line. We do not expect any significant current on the earth connection, other than leakage.

When the energy is carried on three phases only, the nominal rms line voltages refer to the potential between the phases. 34.5 KV rms, for example, is the line voltage in MV lines.

A typical MV to LV transformer configuration is given in Figure 2.29. The three phase line voltage of 34.5 KV MV is connected to the primary windings of a three phase transformer, which is connected in a "Δ" configuration. The secondary terminals are LV terminals, and three windings are now configured in a "Y" form.

In other words, one terminal of each of the secondary windings is connected to earth, while there is no earth connection on the primary. The voltage transformation ratio in these transformers are always stated as the *ratio of line voltages* (i.e. the potential difference between the phases) of primary and secondary windings, although the physical turns ratio of primary and secondary windings correspond to 34.5 KV to 230 V.

Three 220V live lines, neutral and ground are distributed in the buildings through few distribution panels. Precautions against excessive current are taken at each panel. This is for reducing the fire risk in the building and is not useful to avoid electric shock. One can get electric shock either by touching both live and neutral at the same time or by touching live while having contact to ground. The first one is highly unlikely unless one is very careless.

Building floors have a connection to ground reference, although there may be some resistance in between. Therefore if one touches the line while standing on the floor, e.g. with shoes with natural soles (not an isolating sole like rubber), he will get a shock. It is likely that there is an extra precaution at the last panel, where a *residual current device* (RCD) is fitted. This device monitors the leakage current to the ground, and when it exceeds 30 mA, it breaks the circuit. This decreases the severity of the shock.

Ask the lab technician to show and explain the distribution panel in your laboratory.

11. Visit the medium voltage transformer site, which provides energy to your laboratory. Find out the diameter of the cable that delivers the MV energy.

12. Visit a local power plant. Find out what kind of primary energy (i.e. gas, coal, petroleum, wind, hydraulic, etc.) it uses.

13. The voltage reference for the secondary windings is the center tap. We must connect this tap to earth tap in J1. Cut two pieces of 20 cm long *black* colored wire, strip both ends for about 5mm each, and tin them. Join and solder one end of each wire to the center tap of the transformer. Cut two 2 cm long pieces of sleeve. Work two wires into one of the sleeves. Push the sleeve so that the transformer tap is completely covered. Shrink the sleeve. Work one of the wires into remaining sleeve. Solder that wire to earth tap on J1. Push the sleeve and shrink it. Crimp a lead to the other end of the cable and fit it into the center pin of *printed circuit board* (PCB) connector plug, J11. Cut a pair of 10 cm long red wires, strip and tin the ends. Solder one end of each to secondary winding terminals. Fit other ends to side pins of J11, after crimping the leads.

14. Check all connections with a multimeter. Make sure that they correspond to the circuit diagram. Connect the power cable and switch the power ON.

    Using a *mains tester* check if there is mains leakage on the tray.

    Set your multimeter for AC voltage measurement. Connect the leads across the center tap and one of the secondary winding taps. Measure and record the voltage.

This is the rms value of the secondary winding. Calculate the peak value. Measure the voltage across the other winding.

Switch the power OFF.

15. Rest of the power supply circuit is on the PCB. Mount and solder two 30 V varistors. Varistors have high impedance at low voltage levels and low impedance at high voltage levels. When the voltage across it exceeds protection level (30 V in this case), varistor effectively limits the voltage by drawing excessive current.

16. Study the data sheet of PTC thermistors in the Appendix. Determine the rated and switching current of PTC1. What is the I_N, I_S and the on resistance R_N of this thermistor? Record these figures. Mount and solder PTC1 and PTC2. Check the connections. Trim the leads of the PTC's and varistors at the other side of the PCB using a side cutter.

17. The pin configuration of a diode bridge is marked on the package either by the schematic of bridge circuit or by a "+" sign at the pin as marked in the circuit diagram. Mount the bridge correctly and solder it. Trim the leads at the other side of the PCB using a side cutter.

    Connect capacitors C1 and C2. These capacitors are *electrolytic* and have polarity. They contain a *liquid* electrolyte in their case. Either negative pin or positive pin is marked on the capacitor case. Take care to mount them correctly. Solder the capacitors. Check all the connections using a multimeter. Switch the power ON. Measure and record the voltage across C1 and C2 ground pin being common in both cases.

    Switch the power OFF.

18. We use three *voltage regulators* in TRC-10, one regulator for each voltage supply except –8V supply. Voltage regulators are integrated circuits comprising many transistors, diodes etc. All regulators we use in this circuit have three pins: *input*, *output* and *ground*. Voltage regulators convert a rectified and filtered voltage level (like the one in Figure 2.18(c) or Figure 2.19(c)) at their input terminal and convert it into a clean d.c. voltage level *without any ripple*. The two positive supplies 15V and 8V are obtained at the output of two regulators LM7815 and LM7808, respectively. –15V is regulated by LM7915. Voltage regulator requires that the minimum voltage level that appears across its input be about 2V higher that the nominal output voltage, in order to perform regulation. For example, LM7815 requires that the minimum value of the unregulated voltage at its input is 17.5V, in order to provide a regulated nominal 15 V output. Output voltage nominal value has a tolerance. For LM7815, output voltage can be between 14.25 and 15.75 V, regulated. This does not mean that it is allowed to fluctuate between these values, but the level at which the output is fixed can be between these voltages.

    The data sheets of LM78XX and LM79XX series regulators are given in Appendix D. Examine the data sheets. Can you find out the information given above for 7815 in the data sheet? Find out the maximum current that can be drawn from 7815, while regulation is still maintained (peak output current).

Install LM7808 on the PCB and solder it. Make sure that you placed the IC pins correctly on the PCB. Check the connections. Switch the power ON. Measure and record the output voltage, with one decimal unit accuracy. Switch the power OFF.

Install LM7815 on the PCB and solder it. Check the connections. Switch the power ON. Measure and record the output voltage, with one decimal unit accuracy. Switch the power OFF.

Install LM7915 on the PCB and solder it. Check the connections. Switch the power ON. Measure and record the output voltage, with one decimal unit accuracy. Switch the power OFF.

Mount all remaining capacitors, C7, C8 and C9. All of them have polarities. Mount them accordingly. C7, C8 and C9 are *tantalum* capacitors. The electrolyte in tantalum capacitors is in solid form. We include tantalum capacitors to improve the filtering effect at higher frequencies, where the performance of electrolytic capacitors deteriorates.

Check the connections. Switch the power ON. Measure and record all output voltages, with one decimal unit accuracy. Compare these measurements with the previous ones. Switch the power OFF.

19. Mount and solder the protection diodes D12, D14 and D16. These diodes provide a discharge path to capacitors C7, C8 and C9 respectively, when the unregulated voltage input becomes zero. This is a precaution to protect the regulators.

    Mount and solder the protection diodes D13, D15 and D17.

    Check the connections. Switch the power ON. Measure and record all output voltages. Compare these measurements to see if they are the same with the ones in the previous exercise. Switch the power OFF.

20. Find out and record I_N, I_S and the on resistance R_N of PTC3. Mount and solder three PTC thermistors in series with +15 V, -15 V and + 8 V regulator output terminals.

    Check the connections. Switch the power ON. Measure and record the supply voltage levels after the PTC's.

    Connect the multimeter in series with PTC3 as a current meter (not voltmeter). Connect the free end of the current meter to ground. What is the current meter reading? If there were not any PTC on the way, you should read a short circuit current and often the regulator would be burnt out!

    Remove the short circuit and connect the multimeter across D13 as a voltmeter. Record the supply voltage.

21. We also need a –8 V supply in TRC10. The output current requirement on this supply is low. We use a simple circuit containing a zener diode to obtain this voltage (see problem 9).

    Mount and solder resistor R01 and the zener diode D18.

    Check the connections. Switch the power ON. Measure and record the output voltage. The output voltage is V_Z of the diode. Switch the power OFF.

22. The energy provided by the supplies can now be connected to the circuits. We need 20 jumper connections from the power supplies to the circuits of TRC-10. A white straight line between two connection points on the PCB shows each one of these. Locate these jumpers on the PCB. Cut appropriate lengths of wire for each and make the connections by soldering these wires.

2.11. Problems

1. Find R_{eq} in the circuits given below (two significant figures, in Ω, K or M as appropriate):

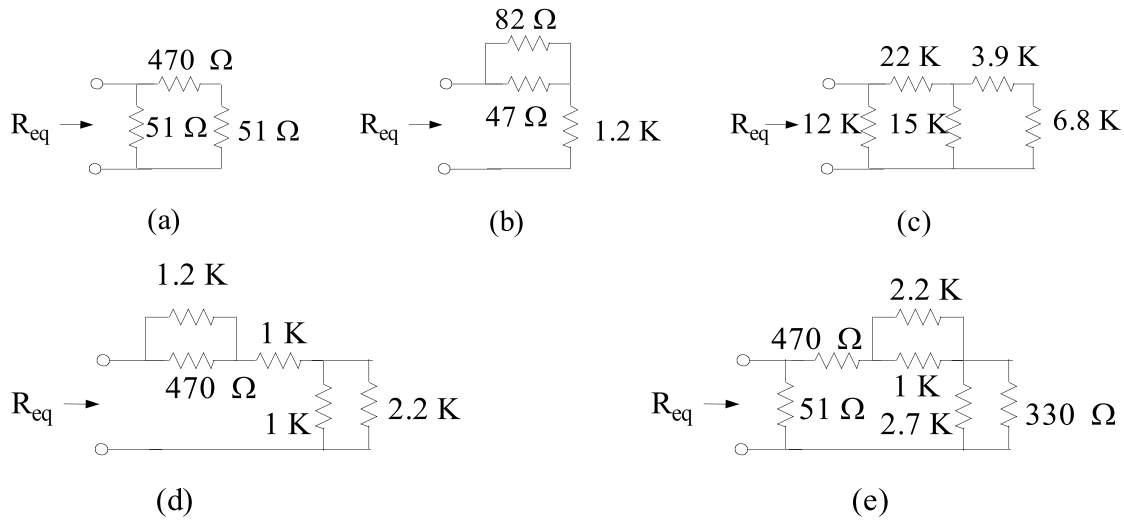


Figure 2.30 Problem 1

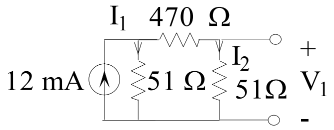
2. Write down the *rms* values of following (two significant figures, in scientific notation):

   (a) 10 cos(1000t) V (b) 1.4 sin(314t+30^o) A (c) 28 cos(ωt+θ) V

3. What are the frequencies of the waveforms in question 2 (three significant figures, in scientific notation)?

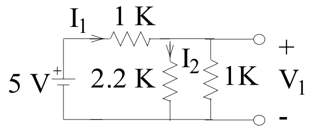
4. A rechargeable Li-ion (*lithium ion*) battery of a mobile phone has nominal voltage of 3.7 V and a capacity of 650 mA-hr. How long a charged battery can supply energy to a 330 Ω resistor, at its rated voltage? Assume that the internal resistance is very small compared to 330 Ω. What is the total energy (in *joules*) delivered to the resistor?

5. Find the marked variables using node analysis in the following circuits (three significant figures, in scientific notation):



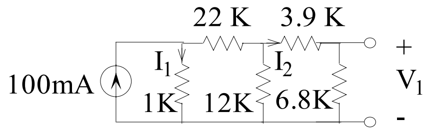
I_1=?; I_2=?; V_1=?

(a)



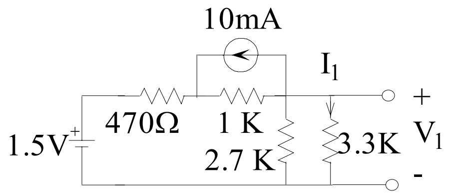
I_1=?; I_2=?; V_1=?

(b)



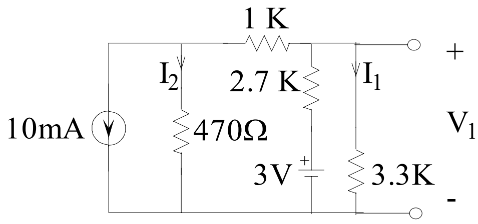
I_1=?; I_2=?; V_1=?

(c)



I_1=?; V_1=?

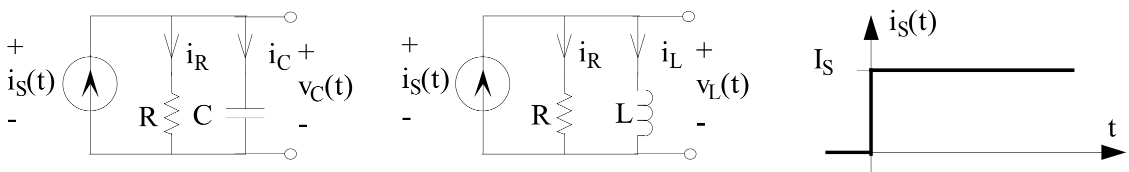
(d)



I_1=?; I_2=?; V_1=?

(e)

6. Consider the circuits (a) and (b) given below. Both circuits are driven by a step current source $i_S(t)$, shown in Figure 2.31. Find and sketch $i_C(t)$, $v_C(t)$, $i_L(t)$, and $v_L(t)$. Assume that $v_C(0) = 0$ and $i_L(0) = 0$.



(a) (b) (c)

Figure 2.31 Circuits for problem 6.

7. Steel reinforced aluminum wires are used in long distance HVOHL. “954 ACSR” wire (954 tells the type of conductor and ACSR stands for “**A**luminum **C**onductor-**S**teel **R**einforced”) has a cross section of 485 mm^2 and a resistance (per unit length) of 0.059 Ω/km. HVOHL are carried by transmission line poles separated by approximately 400 meters, on the average. Considering that the wire is made of aluminum predominantly, calculate the mass of three-phase line between two poles. Calculate the power loss if 32 MW of power is carried over 200 km at a 380 KV_{rms} line. What must be the cross section of the wire to have the same loss over the same distance, if the line voltage is 34.5 KV_{rms}? Calculate the mass for this case.

8. How much energy is stored in C8? What is the amount of charge stored in it?

9. For the –8 V supply circuit formed by R01 and D18, connected across C2 at the bridge rectifier output, calculate the maximum current that can be drawn by external circuits, while maintaining the diode voltage at V_Z = –8 V. Assume that voltage across C2 is approximately –20 V on the average. What is the current drawn from C2 when no external circuit is connected? Recalculate the resistance of R01 (standard resistor value) such that the current delivered to the external circuits is maximized if the maximum power that D18 can dissipate is 0.25 W (D18 burns out if this power limit is exceeded).

10. Find the ratio of R1 to R2 such that the output is –8 V in the circuit given in Figure 2.32. Find a pair of standard resistor values (given in section 2.2) for R1 and R2 such that above ratio is satisfied as much as possible. What is the percent error? Assuming that –8 V is reasonably approximated, can this circuit be used instead of the one in problem 9? Why?

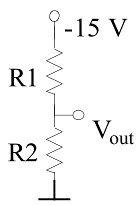


Figure 2.32

11. Consider the amplifier given in the figure, which has input impedance of R_{in}. The voltage gain of the amplifier is 10. Express the voltage gain in dB. What is the power gain when $R_{in} = R_L$? What is the power gain when $R_{in} = 10\ R_L$? In dB?