

Optimal Channel Switching and Randomization over Flat-Fading Channels for Outage Capacity Maximization

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Abstract—Channel switching and parameter randomization can provide performance improvements for communication systems in terms of metrics such as probability of error and channel capacity. In this study, the optimal channel switching and randomization problem is formulated and its solution is characterized for flat-fading Gaussian noise channels with the aim of outage capacity maximization under average power and outage probability constraints. For the single user scenario, it is proved that the optimal solution can always be realized by performing one of the following strategies: (1) Transmission over a single channel with no randomization. (2) Channel switching between two channels with no randomization. (3) Randomization between two parameter sets over a single channel. Hence, the solution can easily be obtained by considering only these three strategies. However, for the multiuser scenario, obtaining the optimal solution can have very high computational complexity. Therefore, an algorithm is proposed to calculate an approximately optimal channel switching and randomization solution (with adjustable approximation accuracy) based on the solution of a linearly constrained linear optimization problem.

Index Terms—Channel switching, outage capacity, time-sharing, power allocation, flat-fading channel.

I. INTRODUCTION

Performance of communication systems can be enhanced via various time-sharing approaches such as channel switching and parameter randomization [1]–[9]. In channel switching, a transmitter and a receiver perform time-sharing among different channels by communicating over only one channel at a given time [1]–[3]. In this way, improvements can be achieved in terms of the average probability of error [1], throughput [10], or channel capacity [2]–[4]. For example, as shown in [1], switching between two channels with a certain time-sharing factor can be necessary in some cases for attaining the minimum average probability of error in an average power constrained binary communication system. In addition, for maximizing the average Shannon capacity between a transmitter and a receiver under average and peak power constraints and in the presence of Gaussian channels, an optimal approach is to implement channel switching between at most two different channels [2].

Apart from channel switching, parameter randomization can also enhance performance of communication systems by employing different parameter values for certain fractions of time; i.e., by performing time-sharing among different parameter sets [5]–[9]. For example, power randomization was carried out in [5] for minimizing the outage probability in a flat block-fading Gaussian channel under an average transmit power constraint. In the context of jamming against digital modulation, the authors of [6] showed that the optimal jamming signal distribution has at most two signal levels along any signaling dimension.

In this letter, we propose the problem of optimal channel switching and randomization for maximizing the outage capacity between a transmitter and a receiver in the presence of average power and outage probability constraints. The channels

between the transmitter and the receiver are modeled as flat-fading and additive Gaussian noise channels, and the channel distribution information (CDI) of each channel is assumed to be available at both the transmitter and the receiver. On the other hand, the channel state information (CSI) of the channels is available only at the receiver. By employing the outage capacity [11] as a performance metric, we derive optimal channel switching and randomization strategies. (The outage capacity is an important and practical metric, based on which communication systems can be designed to support a certain number of users with target data rates.) Main contributions and novelty of this letter can be summarized as follows:

- The problem of optimal channel switching and randomization according to the outage capacity metric is proposed for the first time in the literature. (In [2], the optimal channel switching problem was studied based on the Shannon capacity metric in the absence of randomization or fading. In [5], optimal power randomization was performed for minimizing the outage probability in the absence of channel switching.)
- It is shown (in Proposition 1) that an optimal solution to minimize the average outage capacity can be implemented as one of the three strategies: (1) Transmission over a single channel with no randomization. (2) Channel switching between two channels with no randomization. (3) Randomization between two parameter sets over a single channel.
- For the first time in the literature, we propose and solve the optimal channel switching and randomization problem over flat-fading channels for multiuser systems (i.e., multiple transmitter and receiver pairs) with the aim of maximizing the total outage capacity of users. Unlike those in [2], [4], [5], an optimization theoretic approach is employed to provide a solution in the multiuser scenario.

II. SYSTEM MODEL

We consider the presence of $K \geq 2$ different channels (frequency bands) for communication between a transmitter and a receiver, which can perform channel switching (time-sharing) among these K channels to enhance the capacity of the communication system. As described in [2], during channel switching, only one channel is utilized for the communication between the transmitter and the receiver at any given time, and the transmitter informs the receiver about the occupied channel for synchronization purposes.

In this work, the channels are modeled as flat-fading and additive Gaussian noise channels with various bandwidths and constant power spectral density levels. In particular, for channel i , B_i and $N_i/2$ denote, respectively, the bandwidth and the constant power spectral density level of the additive Gaussian noise, where $i \in \{1, \dots, K\}$. Also, g_i represents the channel gain (i.e., the magnitude square of the channel coefficient) related to channel i between the transmitter and the receiver. It is assumed for each $i \in \{1, \dots, K\}$ that g_i is a continuous random variable, and the support of its probability density function (PDF) is an interval. In addition, g_1, \dots, g_K are modeled as independent random variables. For the information about the channel gains at the transmitter and

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the receiver, it is assumed that the CDI of each channel is available at both the transmitter and the receiver; however, the CSI of the channels is available only at the receiver.¹ In this setting, the outage capacity can be employed as a well-suited performance metric [11].

In addition to channel switching, we also consider randomization of transmit power and outage probability for each channel by considering up to $L \geq 2$ different values per channel. In this regard, we employ *time-sharing factors* denoted by $\lambda_{i,j}$ for $i = 1, \dots, K$ and $j = 1, \dots, L$, which satisfy $\sum_{i=1}^K \sum_{j=1}^L \lambda_{i,j} = 1$ and $\lambda_{i,j} \geq 0$ for all i and j . Namely, $\lambda_{i,j}$ corresponds to the fraction of time when channel i is used with the j th set of parameters (i.e., power level and outage probability pairs) for that channel, which can be denoted as $\theta_{i,j}$ in general. For example, suppose that $K = 3$, $L = 2$, $\lambda_{1,1} = 0.2$, $\lambda_{1,2} = 0.3$, $\lambda_{2,1} = 0.15$, $\lambda_{2,2} = 0$, $\lambda_{3,1} = 0.1$, and $\lambda_{3,2} = 0.25$. Then, over a communication duration of T seconds, channel 1 is used with parameter $\theta_{1,1}$ during the time interval of $[0, 0.2T]$ sec., channel 1 is used with parameter $\theta_{1,2}$ during $[0.2T, 0.5T]$ sec., channel 2 is used with parameter $\theta_{2,1}$ during $[0.5T, 0.65T]$ sec., channel 3 is used with parameter $\theta_{3,1}$ during $[0.65T, 0.75T]$ sec., and channel 3 is used with parameter $\theta_{3,2}$ during $[0.75T, T]$ sec. Channel switching and randomization can provide performance improvements as they facilitate achievement of convex combinations of the performance metric (outage capacity) via time-sharing.

III. MAXIMIZATION OF AVERAGE OUTAGE CAPACITY UNDER AVERAGE POWER AND OUTAGE PROBABILITY CONSTRAINTS

The aim is to perform optimal channel switching and randomization for the maximization of the average outage capacity under average power and outage probability constraints. The outage capacity is defined as the maximum data rate that can be transmitted over a channel with a certain probability of outage (i.e., no proper decoding). Considering the time interval corresponding to the time-sharing factor $\lambda_{i,j}$, the outage capacity of channel i can be expressed as [11]

$$(1 - \varepsilon_{i,j})B_i \log_2(1 + \gamma_{i,j}), \quad (1)$$

where $\varepsilon_{i,j}$ is the outage probability and $\gamma_{i,j}$ denotes the target SNR level below which channel i will be in outage. The outage probability $\varepsilon_{i,j}$ specifies the probability that the SNR level is below the target SNR level $\gamma_{i,j}$, in which case proper decoding cannot be performed. The outage probability can be calculated as [11]

$$\varepsilon_{i,j} = \mathbb{P}\left(\frac{g_i P_{i,j}}{N_i B_i} < \gamma_{i,j}\right) = F_{g_i}\left(\frac{\gamma_{i,j} N_i B_i}{P_{i,j}}\right), \quad (2)$$

where $P_{i,j}$ represents the power level and F_{g_i} denotes the cumulative distribution function (CDF) of g_i .

From (2), the outage capacity in (1) can be stated as

$$C_i(P_{i,j}, \varepsilon_{i,j}) = (1 - \varepsilon_{i,j})B_i \log_2\left(1 + \frac{P_{i,j} F_{g_i}^{-1}(\varepsilon_{i,j})}{N_i B_i}\right), \quad (3)$$

¹The CSI of the channels can be obtained at the receiver based on pilot (training) based channel estimation approaches. Since flat block-fading channels are considered, channel estimation can be performed at the beginning of each block. Also, CDI can be obtained based on previous channel estimates and/or statistical channel models related to the operating environment. In particular, by utilizing previous channel estimates, the receiver can form statistical knowledge about the channel parameters, and share this knowledge with the transmitter via a feedback mechanism.

where $F_{g_i}^{-1}$ denotes the inverse CDF of g_i . For channel i , $P_{i,j}$ and $\varepsilon_{i,j}$ (or, equivalently, $\gamma_{i,j}$) can be regarded as design parameters that are subject to power limits and acceptable levels of outage probability, respectively. Accordingly, the optimal channel switching and randomization problem for outage capacity maximization is proposed as follows:

$$\max_{\{\lambda_{i,j}, P_{i,j}, \varepsilon_{i,j}\}_{i,j=1}^{K,L}} \sum_{i=1}^K \sum_{j=1}^L \lambda_{i,j} C_i(P_{i,j}, \varepsilon_{i,j}) \quad (4a)$$

$$\text{subject to } \sum_{i=1}^K \sum_{j=1}^L \lambda_{i,j} P_{i,j} \leq P_{\text{av}}, \quad (4b)$$

$$\sum_{i=1}^K \sum_{j=1}^L \lambda_{i,j} \varepsilon_{i,j} \leq \varepsilon_{\text{av}}, \quad (4c)$$

$$P_{i,j} \in [0, P_{\text{pk}}], \quad \forall i \in \mathcal{S}_c, \quad \forall j \in \mathcal{S}_r, \quad (4d)$$

$$\varepsilon_{i,j} \in [0, \varepsilon_{\text{pk}}], \quad \forall i \in \mathcal{S}_c, \quad \forall j \in \mathcal{S}_r, \quad (4e)$$

$$\sum_{i=1}^K \sum_{j=1}^L \lambda_{i,j} = 1, \quad (4f)$$

$$\lambda_{i,j} \geq 0, \quad \forall i \in \mathcal{S}_c, \quad \forall j \in \mathcal{S}_r, \quad (4g)$$

where $\mathcal{S}_c \triangleq \{1, \dots, K\}$, $\mathcal{S}_r \triangleq \{1, \dots, L\}$, P_{av} is the average power constraint, ε_{av} is the average outage probability constraint, P_{pk} is the peak power constraint, and ε_{pk} is peak outage probability constraint. Due to practical reasons, it is assumed that $P_{\text{av}} < P_{\text{pk}}$. Although the problem in (4) is a challenging *non-convex* optimization problem over a $3KL$ -dimensional space in general, we characterize its solution, denoted by $\{\lambda_{i,j}^*, P_{i,j}^*, \varepsilon_{i,j}^*\}_{i,j=1}^{K,L}$, in the following proposition.

Proposition 1: Consider the following problem:

$$\max_{\{\nu, P_1, P_2, \varepsilon_1, \varepsilon_2\}} \nu C_{\text{max}}(P_1, \varepsilon_1) + (1 - \nu) C_{\text{max}}(P_2, \varepsilon_2) \quad (5a)$$

$$\text{subject to } \nu P_1 + (1 - \nu) P_2 \leq P_{\text{av}}, \quad (5b)$$

$$\nu \varepsilon_1 + (1 - \nu) \varepsilon_2 \leq \varepsilon_{\text{av}}, \quad (5c)$$

$$P_1 \in [0, P_{\text{pk}}], \quad P_2 \in [0, P_{\text{pk}}], \quad (5d)$$

$$\varepsilon_1 \in [0, \varepsilon_{\text{pk}}], \quad \varepsilon_2 \in [0, \varepsilon_{\text{pk}}], \quad \nu \in [0, 1], \quad (5e)$$

$$\text{where } C_{\text{max}}(P, \varepsilon) = \max_{i \in \mathcal{S}_c} C_i(P, \varepsilon). \quad (6)$$

Let $(\nu^*, P_1^*, P_2^*, \varepsilon_1^*, \varepsilon_2^*)$ denote the solution of (5), and let ℓ and m be defined as

$$\ell = \arg \max_{i \in \mathcal{S}_c} C_i(P_1^*, \varepsilon_1^*), \quad m = \arg \max_{i \in \mathcal{S}_c} C_i(P_2^*, \varepsilon_2^*). \quad (7)$$

Then, the solution of (4) can be specified as one of the following strategies:

- **Conventional Strategy– Single Channel with no Randomization:** If $\nu^* = 1$ or if $\ell = m$ and $(P_1^*, \varepsilon_1^*) = (P_2^*, \varepsilon_2^*)$, then a solution of (4) can be stated as $\lambda_{\ell,1}^* = 1$, $P_{\ell,1}^* = P_1^*$, $\varepsilon_{\ell,1}^* = \varepsilon_1^*$, and $\lambda_{i,j}^* = 0$ for all $(i,j) \neq (\ell,1)$.² Similarly, if $\nu^* = 0$, a solution of (4) can be stated as $\lambda_{m,1}^* = 1$, $P_{m,1}^* = P_2^*$, $\varepsilon_{m,1}^* = \varepsilon_2^*$, and $\lambda_{i,j}^* = 0$ for all $(i,j) \neq (m,1)$. Namely, one of the channels is used exclusively without any randomization.

- **CS2 Strategy– Channel Switching between Two Channels with no Randomization:** If $\ell \neq m$ and $\nu^* \in (0, 1)$, then a solution of (4) can be stated as $\lambda_{\ell,1}^* = \nu^*$, $P_{\ell,1}^* = P_1^*$,

²It is noted that the second index in the subscripts is chosen as 1 arbitrarily as it does not affect the performance of the solution.

$\varepsilon_{\ell,1}^* = \varepsilon_1^*$, $\lambda_{m,1}^* = 1 - \nu^*$, $P_{m,1}^* = P_2^*$, $\varepsilon_{m,1}^* = \varepsilon_2^*$, and $\lambda_{i,j}^* = 0$ for all $(i,j) \notin \{(\ell,1), (m,1)\}$. That is, channel switching is performed between two channels without any randomization over each channel.

- **Rand2 Strategy– Randomization between two Power-Outage Probability pairs over a Single Channel:** If $\ell = m$, $\nu^* \in (0,1)$, and $(P_1^*, \varepsilon_1^*) \neq (P_2^*, \varepsilon_2^*)$, then a solution of (4) can be stated as $\lambda_{\ell,1}^* = \nu^*$, $P_{\ell,1}^* = P_1^*$, $\varepsilon_{\ell,1}^* = \varepsilon_1^*$, $\lambda_{\ell,2}^* = 1 - \nu^*$, $P_{\ell,2}^* = P_2^*$, $\varepsilon_{\ell,2}^* = \varepsilon_2^*$, and $\lambda_{i,j}^* = 0$ for all $(i,j) \notin \{(\ell,1), (\ell,2)\}$.³ That is, randomization (time-sharing) is performed between two different parameter sets over a single channel (i.e., without channel switching).

Proof: By introducing a vector variable as $\theta_{i,j} = [P_{i,j}, \varepsilon_{i,j}]$, the problem in (4) can be stated as

$$\max_{\{\lambda_{i,j}, \theta_{i,j}\}_{i,j=1}^{K,L}} \sum_{i=1}^K \sum_{j=1}^L \lambda_{i,j} C_i(\theta_{i,j}) \quad (8a)$$

$$\text{subject to } \sum_{i=1}^K \sum_{j=1}^L \lambda_{i,j} \theta_{i,j} \leq [P_{\text{av}}, \varepsilon_{\text{av}}], \quad (8b)$$

$$\theta_{i,j} \in \mathcal{T}, \quad \forall i \in \mathcal{S}_c, \quad \forall j \in \mathcal{S}_r, \quad (8c)$$

$$\sum_{i=1}^K \sum_{j=1}^L \lambda_{i,j} = 1, \quad \lambda_{i,j} \geq 0, \quad \forall i \in \mathcal{S}_c, \quad \forall j \in \mathcal{S}_r, \quad (8d)$$

where $\mathcal{T} \triangleq [0, P_{\text{pk}}] \times [0, \varepsilon_{\text{pk}}]$. To characterize the solution of (8), we first present the following problem:

$$\max_{p(\theta)} \int C_{\text{max}}(\theta) p(\theta) d\theta \quad (9a)$$

$$\text{subject to } \int \theta p(\theta) d\theta \leq [P_{\text{av}}, \varepsilon_{\text{av}}], \quad (9b)$$

$$\theta \in \mathcal{T}, \quad \int p(\theta) d\theta = 1, \quad p(\theta) \geq 0, \quad \forall \theta \quad (9c)$$

where $C_{\text{max}}(\theta_{i,j}) = \max_{i \in \mathcal{S}_c} C_i(\theta_{i,j})$ as in (6), and $p(\theta)$ denotes the PDF of θ . The problem in (9) provides an upper bound on (8) since it employs the maximum of C_i 's in its objective function and more generic weighting coefficients.

The problems in the form of (9) have been investigated in various contexts in the literature; e.g., [7]–[9]. By adopting a similar approach, we define set \mathcal{U} and set \mathcal{W} as follows: $\mathcal{U} = \{(\theta, C_{\text{max}}(\theta)) \text{ for all } \theta \in \mathcal{T}\}$ and $\mathcal{W} = \{(\int \theta p(\theta) d\theta, \int C_{\text{max}}(\theta) p(\theta) d\theta) \text{ for all } \int p(\theta) d\theta = 1, p(\theta) \geq 0, \theta \in \mathcal{T}\}$. It is noted that \mathcal{W} contains the solution of (9) since it consists of all possible values of $\int C_{\text{max}}(\theta) p(\theta) d\theta$ and $\int \theta p(\theta) d\theta$ subject to the constraints in (9c). Also, via the arguments in [7]–[9], it can be shown that \mathcal{W} is equal to the convex hull of \mathcal{U} ; i.e., $\mathcal{W} = \text{hull}(\mathcal{U})$. Therefore, as a result of Carathéodory's theorem [12], [13], any element of \mathcal{W} can be expressed as a convex combination of $\dim(\mathcal{U}) + 1 = 3$ elements in \mathcal{U} , where $\dim(\mathcal{U}) = 2$ since $\mathcal{U} \subset \mathbb{R}^2$. In addition, as the maximizer of (9) must reside on the boundary of $\text{hull}(\mathcal{U})$, the solution of (9) can be achieved by a convex combination of $\dim(\mathcal{U}) = 2$ elements in \mathcal{U} by Carathéodory's theorem [12], [13]. Hence, an optimal $p(\theta)$ can be specified as $p(\theta) = \nu \delta(\theta - \theta_1) + (1 - \nu) \delta(\theta - \theta_2)$, where $\delta(\cdot)$ denotes the Dirac delta function. By inserting this specific $p(\theta)$ expression into (9), we obtain the following problem:

$$\max_{\{\nu, \theta_1, \theta_2\}} \nu C_{\text{max}}(\theta_1) + (1 - \nu) C_{\text{max}}(\theta_2) \quad (10a)$$

³The second indices in the subscripts are chosen as 1 and 2 arbitrarily.

$$\text{subject to } \nu \theta_1 + (1 - \nu) \theta_2 \leq [P_{\text{av}}, \varepsilon_{\text{av}}], \quad (10b)$$

$$\theta_1 \in \mathcal{T}, \quad \theta_2 \in \mathcal{T}, \quad \nu \in [0, 1], \quad (10c)$$

which is guaranteed to achieve the same maximum value as (9). It is noted that the problem in (10) is the same as (5) in Proposition 1 as $\theta_1 = [P_1, \varepsilon_1]$ and $\theta_2 = [P_2, \varepsilon_2]$. Let $(\nu^*, \theta_1^*, \theta_2^*)$ denote the solution of (10), and let ℓ and m be defined as in (7), where $\theta_1^* = [P_1^*, \varepsilon_1^*]$ and $\theta_2^* = [P_2^*, \varepsilon_2^*]$. The maximum value of (10) (equivalently, of (9)) can be achieved by the problem in (4) via the conventional strategy, the CS2 strategy, or the Rand2 strategy, as specified in the proposition. Since the problem in (9) is an upper bound on the problem in (4), and the maximum value of (9) can be achieved by (4) via the conventional, CS2, or Rand2 strategies, it is concluded that the solution of (4) can be characterized by the strategies specified in the proposition. ■

Based on Proposition 1, the solution of (4) can be obtained by searching over three possible strategies, which significantly reduces the computation complexity of the problem. Namely, instead of a search over a $3KL$ -dimensional space, the optimal solution can be obtained via a four-dimensional search as specified in the following: First, it is noted that $C_{\text{max}}(P, \varepsilon)$ in (6) is a monotone increasing function of $P > 0$ for any $\varepsilon \in [0, 1]$ since each $C_i(P, \varepsilon)$ in (3) is monotone increasing with respect to P . Therefore, any approach with $\nu P_1 + (1 - \nu) P_2 < P_{\text{av}}$ cannot be a solution of (5) since it can always be improved by increasing at least one of the power levels (as $P_{\text{av}} < P_{\text{pk}}$). Hence, $\nu P_1 + (1 - \nu) P_2 = P_{\text{av}}$ must be satisfied in (5). By utilizing this equality, the problem in (5) can be simplified as

$$\max_{\{P_1, P_2, \varepsilon_1, \varepsilon_2\}} \frac{P_{\text{av}} - P_2}{P_1 - P_2} C_{\text{max}}(P_1, \varepsilon_1) + \frac{P_1 - P_{\text{av}}}{P_1 - P_2} C_{\text{max}}(P_2, \varepsilon_2) \quad (11a)$$

$$\text{subject to } \frac{P_{\text{av}} - P_2}{P_1 - P_2} \varepsilon_1 + \frac{P_1 - P_{\text{av}}}{P_1 - P_2} \varepsilon_2 \leq \varepsilon_{\text{av}}, \quad (11b)$$

$$P_1 \in (P_{\text{av}}, P_{\text{pk}}], \quad P_2 \in [0, P_{\text{av}}], \quad (11c)$$

$$\varepsilon_1 \in [0, \varepsilon_{\text{pk}}], \quad \varepsilon_2 \in [0, \varepsilon_{\text{pk}}]. \quad (11d)$$

Let $(P_1^*, P_2^*, \varepsilon_1^*, \varepsilon_2^*)$ denote the solution of (11). Then, ν^* is obtained as $\nu^* = (P_{\text{av}} - P_2^*) / (P_1^* - P_2^*)$. Also, ℓ and m are calculated as in (7). Then, the solution of (4) corresponds to the use of the conventional, CS2, or Rand2 strategies, as specified in Proposition 1 based on the parameters ν^* , P_1^* , P_2^* , ε_1^* , ε_2^* , ℓ , and m . (To specify the computational complexity of solving (11) via exhaustive search, suppose that ε_1 and ε_2 are discretized with a step size of Δ_0 , and P_1 and P_2 are discretized with step sizes of Δ_1 and Δ_2 , respectively. Then, the objective function in (11) should be evaluated about $\varepsilon_{\text{pk}}^2 P_{\text{av}} (P_{\text{pk}} - P_{\text{av}}) / (\Delta_0^2 \Delta_1 \Delta_2)$ times to find the solution.)

Remark 1: Proposition 1 also implies that the use of both channel switching and randomization is not needed to achieve the solution of (4). In other words, when optimal channel switching is performed as in the CS2 strategy, randomization over any of the channels does not bring any additional benefits. Similarly, when the optimal solution is achieved via randomization over a single channel as in the Rand2 strategy, the average outage capacity cannot be increased further via channel switching.

IV. EXTENSION TO MULTIUSER SYSTEMS

In this section, we consider a multiuser system with U transmitter-receiver pairs, i.e., users, which aim to communicate over the K available channels with the ability of performing channel switching and randomization. The aim

is to maximize the total outage capacity of the users under average power and outage probability constraints by jointly optimizing the parameters of all the users in a centralized fashion. Accordingly, the following problem is proposed (cf. (4)):

$$\max_{\{\lambda_{i,j}^{(u)}, P_{i,j}^{(u)}, \varepsilon_{i,j}^{(u)}\}_{i,j,u=1}^{K,L,U}} \sum_{u=1}^U \sum_{i=1}^K \sum_{j=1}^L \lambda_{i,j}^{(u)} C_i^{(u)}(P_{i,j}^{(u)}, \varepsilon_{i,j}^{(u)}) \quad (12a)$$

$$\text{subject to } \sum_{i=1}^K \sum_{j=1}^L \lambda_{i,j}^{(u)} P_{i,j}^{(u)} \leq P_{\text{av}}^{(u)}, \quad \forall u \in \mathcal{S}_u, \quad (12b)$$

$$\sum_{i=1}^K \sum_{j=1}^L \lambda_{i,j}^{(u)} \varepsilon_{i,j}^{(u)} \leq \varepsilon_{\text{av}}^{(u)}, \quad \forall u \in \mathcal{S}_u, \quad (12c)$$

$$P_{i,j}^{(u)} \in [0, P_{\text{pk}}^{(u)}], \quad \forall i \in \mathcal{S}_c, \quad \forall j \in \mathcal{S}_r, \quad \forall u \in \mathcal{S}_u, \quad (12d)$$

$$\varepsilon_{i,j}^{(u)} \in [0, \varepsilon_{\text{pk}}^{(u)}], \quad \forall i \in \mathcal{S}_c, \quad \forall j \in \mathcal{S}_r, \quad \forall u \in \mathcal{S}_u, \quad (12e)$$

$$\sum_{i=1}^K \sum_{j=1}^L \lambda_{i,j}^{(u)} \leq 1, \quad \forall u \in \mathcal{S}_u, \quad (12f)$$

$$\lambda_{i,j}^{(u)} \geq 0, \quad \forall i \in \mathcal{S}_c, \quad \forall j \in \mathcal{S}_r, \quad \forall u \in \mathcal{S}_u, \quad (12g)$$

$$\sum_{u=1}^U \sum_{j=1}^L \lambda_{i,j}^{(u)} \leq 1, \quad \forall i \in \mathcal{S}_c, \quad (12h)$$

where $\mathcal{S}_u \triangleq \{1, \dots, U\}$, and the capacity function and the parameters are defined as in (4) with the addition of superscript (u) for denoting the user index. The constraint in (12h) is required in multiuser scenarios to make sure that the total time-sharing factor over each channel does not exceed one.

The problem in (12) is very challenging to solve in general since its solution cannot be reduced to a set of three strategies as in Section III and exhaustive search has prohibitive complexity. However, we can employ a convex relaxation approach [7], [14] to approximate the non-convex problem in (12) with a convex problem with the ability to adjust the approximation accuracy. To this aim, instead of the continuum of values for the $P_{i,j}^{(u)}$ and $\varepsilon_{i,j}^{(u)}$ terms, we consider a set of N_u possible (known) values for them specified as $(P_{i,j}^{(u)}, \varepsilon_{i,j}^{(u)}) \in \{(\tilde{P}_1^{(u)}, \tilde{\varepsilon}_1^{(u)}), \dots, (\tilde{P}_{N_u}^{(u)}, \tilde{\varepsilon}_{N_u}^{(u)})\}$ for each $u \in \mathcal{S}_u$. Accordingly, we define the following vectors:

$$\tilde{\lambda} = [\tilde{\lambda}^{(1)} \dots \tilde{\lambda}^{(U)}] \quad \text{and} \quad \tilde{\mathbf{C}} = [\tilde{\mathbf{C}}^{(1)} \dots \tilde{\mathbf{C}}^{(U)}], \quad (13)$$

$$\text{where } \tilde{\lambda}^{(u)} = [\lambda_{1,1}^{(u)} \dots \lambda_{1,N_u}^{(u)} \dots \lambda_{K,1}^{(u)} \dots \lambda_{K,N_u}^{(u)}], \quad (14)$$

$$\tilde{\mathbf{C}}^{(u)} = [C_1^{(u)}(\tilde{P}_1^{(u)}, \tilde{\varepsilon}_1^{(u)}) \dots C_1^{(u)}(\tilde{P}_{N_u}^{(u)}, \tilde{\varepsilon}_{N_u}^{(u)}) \dots \dots \dots C_K^{(u)}(\tilde{P}_1^{(u)}, \tilde{\varepsilon}_1^{(u)}) \dots C_K^{(u)}(\tilde{P}_{N_u}^{(u)}, \tilde{\varepsilon}_{N_u}^{(u)})] \quad (15)$$

for $u \in \mathcal{S}_u$. Based on these definitions, (12) is approximated as

$$\max_{\tilde{\lambda}} \tilde{\mathbf{C}}^T \tilde{\lambda} \quad (16a)$$

$$\text{subject to } \mathbf{R} \tilde{\lambda} \preceq [P_{\text{av}}^{(1)} \dots P_{\text{av}}^{(U)}]^T, \quad (16b)$$

$$\mathbf{E} \tilde{\lambda} \preceq [\varepsilon_{\text{av}}^{(1)} \dots \varepsilon_{\text{av}}^{(U)}]^T, \quad (16c)$$

$$\mathbf{B} \tilde{\lambda} \preceq \mathbf{1}_U, \quad \tilde{\lambda} \succeq \mathbf{0}, \quad \mathbf{G} \tilde{\lambda} \preceq \mathbf{1}_K, \quad (16d)$$

with $\mathbf{1}_X$ representing a column vector of ones with X elements, $\mathbf{R} = \text{blkdiag}(\mathbf{1}_K^T \otimes \tilde{\mathbf{P}}^{(1)}, \dots, \mathbf{1}_K^T \otimes \tilde{\mathbf{P}}^{(U)})$, $\mathbf{E} = \text{blkdiag}(\mathbf{1}_K^T \otimes \tilde{\varepsilon}^{(1)}, \dots, \mathbf{1}_K^T \otimes \tilde{\varepsilon}^{(U)})$, $\mathbf{B} =$

$\text{blkdiag}(\mathbf{1}_{KN_1}^T, \dots, \mathbf{1}_{KN_U}^T)$, and $\mathbf{G} = [\mathbf{I}_K \otimes \mathbf{1}_{N_1}^T \dots \mathbf{I}_K \otimes \mathbf{1}_{N_U}^T]$, where blkdiag specifies a block diagonal matrix constructed by the given matrices, \otimes denotes the Kronecker product, \mathbf{I}_K is the $K \times K$ identity matrix, $\tilde{\mathbf{P}}^{(u)} = [\tilde{P}_1^{(u)} \dots \tilde{P}_{N_u}^{(u)}]$, and $\tilde{\varepsilon}^{(u)} = [\tilde{\varepsilon}_1^{(u)} \dots \tilde{\varepsilon}_{N_u}^{(u)}]$ for $u \in \mathcal{S}_u$.

It is noted that (16) is a linearly constrained linear optimization problem. Therefore, it can be solved rapidly via linear or convex optimization algorithms such as the simplex or the interior point method [14]. Although (16) is an approximation to (12), the approximation accuracy can be enhanced by increasing the number of possible values for the $(P_{i,j}^{(u)}, \varepsilon_{i,j}^{(u)})$ pairs, i.e., by increasing N_1, \dots, N_U , with the cost of higher computational complexity. (It is noted that the problem in (16) has polynomial complexity in the number of variables, which is equal to $K \sum_{u=1}^U N_u$ [14].)

Remark 2: It is also possible to add fairness constraints to (12) in the form of $\sum_{i=1}^K \sum_{j=1}^L \lambda_{i,j}^{(u)} C_i^{(u)}(P_{i,j}^{(u)}, \varepsilon_{i,j}^{(u)}) \geq \varsigma_u \quad \forall u \in \mathcal{S}_u$. In that case, the solution can be obtained by an approximate problem as in (16) by adding linear constraints related to fairness.

V. NUMERICAL RESULTS AND CONCLUSIONS

In this section, numerical examples are presented to corroborate the theoretical results by considering two settings. In the first setting, as in [2], we consider $K = 3$ channels with the following bandwidths and noise levels: $B_1 = 1$ MHz, $B_2 = 5$ MHz, $B_3 = 10$ MHz, $N_1 = 10^{-12}$ W/Hz, $N_2 = 10^{-11}$ W/Hz, and $N_3 = 10^{-11}$ W/Hz. In this setting, there can exist one user or two users in the system. We model the channels as independent Rayleigh fading with the channel gains being exponentially distributed with the following CDFs and PDFs: $F_{g_i^{(u)}}(g) = 1 - e^{-\alpha_i^{(u)} g}$ and $p_{g_i^{(u)}}(g) = \alpha_i^{(u)} e^{-\alpha_i^{(u)} g}$ if $g \geq 0$ for $i \in \{1, 2, 3\}$ and $u \in \{1, 2\}$. The parameters are given by $\alpha_1^{(1)} = 0.25$, $\alpha_2^{(1)} = 1$, and $\alpha_3^{(1)} = 0.75$ for user 1, and $\alpha_1^{(2)} = 0.95$, $\alpha_2^{(2)} = 0.35$, and $\alpha_3^{(2)} = 0.3$ for user 2. Also, the peak power and peak outage constraints are set as $P_{\text{pk}} = 10P_{\text{av}}$ and $\varepsilon_{\text{pk}} = 10\varepsilon_{\text{av}}$.

We first assume that only user 1 exists in the system and investigate the outage capacity maximization problem in (4), the solution of which is specified by Proposition 1. In Fig. 1, the average outage capacity achieved by the solution of (4) (which is obtained via (11)) is plotted with respect to the average power constraint, P_{av} for $\varepsilon_{\text{av}} = 0.01$ (labeled as ‘Proposed (single user)’ in the figure). In addition, the average outage capacities achieved by the conventional strategy, which employs the best channel all the time at the average power limit (and with the corresponding optimal outage probability) are presented for comparison purposes (labeled as ‘Conventional (single user)’).⁴ It is observed from Fig. 1 that employing the best channel all the time (i.e., conventional strategy) is not always optimal, in accordance with Proposition 1. For example, when $P_{\text{av}} = 10^{-2}$ mW, the proposed (optimal) solution employs channel 1 with a time-sharing factor of 0.33444 and a power level of 0.029901 mW (corresponding to an outage probability of 0.029901) and does not send any power in the remaining duration (i.e., employs zero power with a time-sharing factor of 0.66556). This can be considered as a special case of the Rand2 Strategy (or, CS2 Strategy) in Proposition 1 with $P_2^* = 0$, which results in an average outage

⁴This corresponds to solving (5) over ε_1 by setting $\nu = 1$ and $P_1 = P_{\text{av}}$.

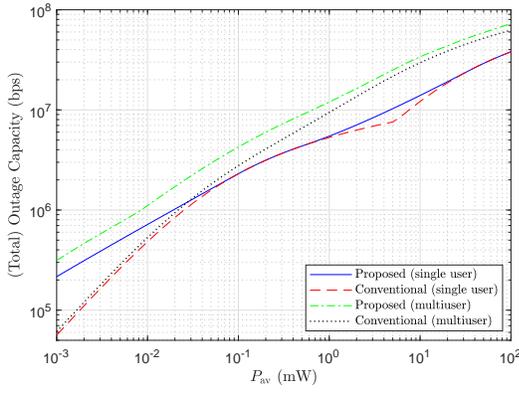


Fig. 1. Average outage capacity versus P_{av} achieved by the solution of (4) (labeled as ‘Proposed (single user)’) and by the Conventional Strategy in Section III for the single user scenario. Also, the total average outage capacity is plotted versus P_{av} for the multiuser scenario with $U = 2$ users considering both the proposed approach based on (16) and the conventional approach.

capacity of 0.71742 Mbps. (Such an on-off strategy can never be optimal according to the Shannon capacity metric in [2] due to its concavity.) As another example, when $P_{av} = 5$ mW, the proposed (optimal) solution corresponds to the CS2 Strategy, which uses channel 1 with a time-sharing factor of 0.7023 and a power level of 1.5561 mW, and channel 3 with a time-sharing factor of 0.2977 and a power level of 13.125 mW, leading to an average outage capacity of 10.292 Mbps. However, for the same setting, the conventional strategy employs channel 1 exclusively, and achieves an outage capacity of 7.5817 Mbps. On the other hand, when $P_{av} = 100$ mW, the proposed strategy corresponds to the conventional strategy, which utilizes channel 3 exclusively. It is also important to mention that the turning points in Fig. 1 occur when a strategy starts employing a different channel.

Next, we consider the presence of both user 1 and user 2 in the system described above, and obtain the proposed channel switching and randomization strategy in Section IV. To implement the proposed strategy, we first generate a set of N_u possible values of $(P_{i,j}^{(u)}, \varepsilon_{i,j}^{(u)})$ for $u \in \{1, 2\}$ by forming a vector of power levels consisting of 51 values between 0 and P_{pk} (with equal spacing) and similarly a vector of outage probabilities consisting of 51 values between 0 and ε_{pk} (hence, $N_u = 2601$ for $u \in \{1, 2\}$).⁵ Then, the proposed strategy is obtained by solving (16) (labeled as ‘Proposed (multiuser)’ in Fig. 1). For comparison purposes, we also illustrate the conventional strategy in Fig. 1 (labeled as ‘Conventional (multiuser)’), which performs sequential assignments of users to channels over which they achieve the highest outage capacities by considering all possible assignment orders for the users. (In the conventional strategy, at most one user is assigned to a channel, and the channel is completely utilized by a user once it is assigned; i.e., no time-sharing.) It is observed from Fig. 1 that the proposed approach for the multiuser scenario outperforms the conventional strategy for all values of P_{av} , especially for low P_{av} ’s.

Finally, we consider the second setting with $K = 5$ channels, which have the following bandwidths and noise levels: $B_1 = 1$ MHz, $B_2 = 5$ MHz, $B_3 = 10$ MHz, $B_4 = 15$ MHz, $B_5 = 20$ MHz, $N_1 = 10^{-12}$ W/Hz, $N_2 = 10^{-11}$ W/Hz, $N_3 = 10^{-11}$ W/Hz, $N_4 = 10^{-12}$ W/Hz, and $N_5 = 10^{-11}$ W/Hz. The number of users, U , is varied from 1 to 10. The channel gain parameters for the users (i.e., $\alpha_i^{(u)}$ ’s) are generated uniformly

⁵Increasing N_u further does not lead to notable performance improvement.

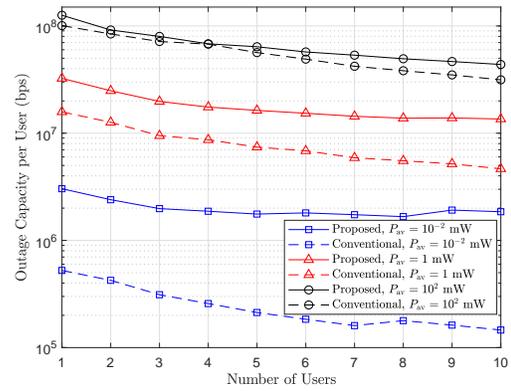


Fig. 2. Average outage capacity per user versus the number users for the proposed approach based on (16) and the conventional approach, where $K = 5$ and $\varepsilon_{av} = 0.01$.

and independently over $[0.1, 1]$ in MATLAB with seed 1 (namely, “0.1 + 0.9 * rand(10, 5)”). As in the first setting, $P_{pk} = 10P_{av}$ and $\varepsilon_{pk} = 10\varepsilon_{av}$. Also, the N_u possible values of $(P_{i,j}^{(u)}, \varepsilon_{i,j}^{(u)})$ are generated as in the previous paragraph for each u . In Fig. 2, the average outage capacity per user is plotted versus the number of users for three different values of P_{av} , where $\varepsilon_{av} = 0.01$. It is observed that the proposed strategy based on (16) outperforms the conventional strategy in all cases, especially for low values of P_{av} . It is also noted that as the number of users increases, the average outage capacity per user tends to decrease in general since there exist limited resources. However, this trend is not monotone since the channel parameters of the users are generated randomly (i.e., a new user with favorable channel characteristics may improve the average outage capacity per user).

As an important direction for future work, channel switching delays can be considered when determining the optimal channel switching and randomization strategies.

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