Technical Report

Derivation of Users' Received Signal Power in Cell On Edge Configured HetNet Communication System

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I. Introduction

In this report, the analytical formulas for cumulative distribution of received power (P_r) for the UEs in the heterogeneous network model given in [1], will be derived by using a geometrical approach. This approach is valid when Macro and Micro BSs are located at fixed locations. As expressed in [1], we assume Cell On Edge configuration with a fixed number of micro base stations (BS). Using the distribution of received power, the distribution of data rate is obtained in [1].

II. Modeling Coverage Area of Micro Base Stations

The range extended coverage (yellow region) of Micro BS for $B > 0dB$ is shown in Figure 1. This is the region for which $P_{r,Micro}10^{\frac{B}{10}} > P_{r,Macro}$, where $P_{r,Micro}$ is the power received from the closest Micro BS, $P_{r,Macro}$ is the received power from Macro BS and B is the BIAS parameter.

Assuming Macro BS is located at point $(0,0)$ and Micro BS is located at $(d_{Micro}, 0)$ any point having coordinates (x, y) on the contour of Cell Range Extended coverage region should satisfy

$$\frac{P_1}{(x^2+y^2)^{\frac{\gamma_1}{2}}} = 10^{\frac{B}{10}} \frac{P_2}{((x-d_{Micro})^2+y^2)^{\frac{\gamma_2}{2}}}. \tag{1}$$

(1) is numerically solved for a given value of B and coverage of Micro BS is obtained. In order to simplify the analytical calculations, these coverage regions are approximated by circles centered at points $c = p_3 e^{j\frac{2\pi i}{N_{MICRO}}}$, $i = 0, 1, 2...(N_{MICRO}-1)$ and have the radius of $r = \frac{|p_2-p_1|}{2}$. Here, $p_3 = \frac{p_1+p_2}{2}$ is the point at coordinates $(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2})$ and p_1, p_2 are the points where coverage contour given by eq. (1), of Micro BS centered at p_4 intersects with the line passing through points $C_1(0,0)$ and p_4.

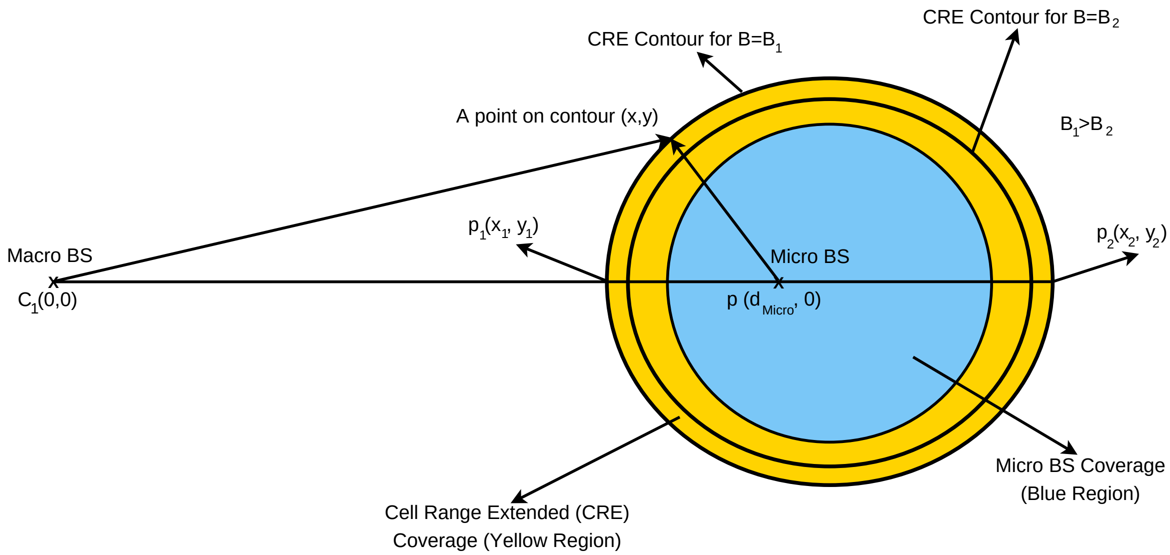


Fig. 1: CRE region contour for $B > 0dB$

The parameter ν is introduced to validate the circle approximation and it is defined as $\nu = \frac{S_1}{S_2}$, where S_1 is the intersection area of the actual base station coverage with approximated base station coverage and S_2 is the union of these two areas. Table I lists the variation of ν with B. From Table I it can be seen that $\nu > 95\%$ for all B values and the model is more accurate for small B values. In addition to Table I, Figures 2a and 2b show how this approximation is close to the actual coverage region for $B = 10dB$ and $B = 20dB$.

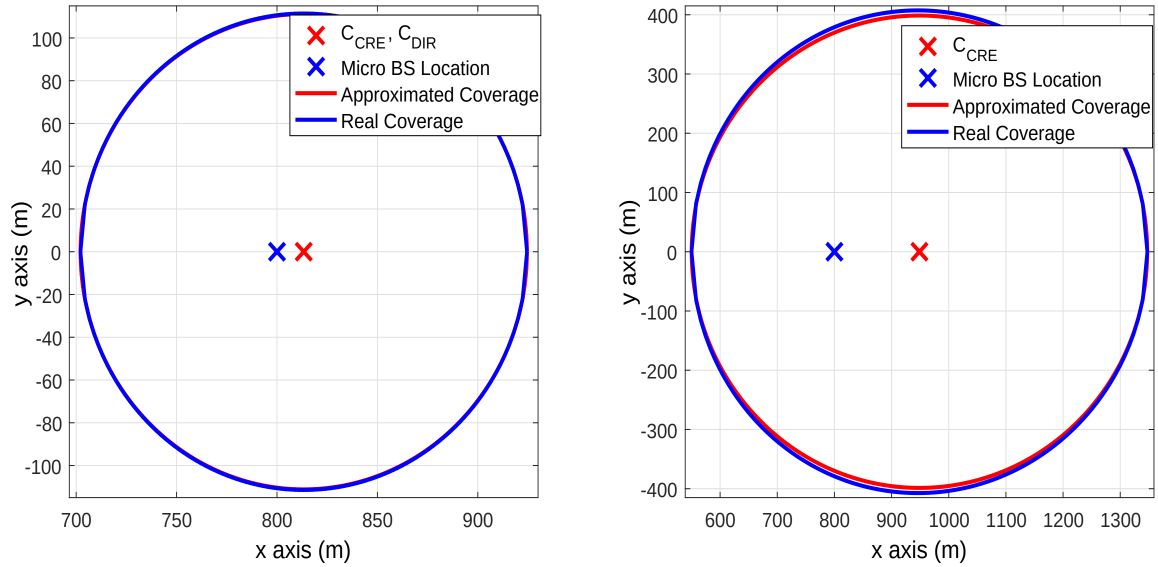
TABLE I: Variation of ν With B

B	0	5	10	15	20	25
ν	0.9979	0.9963	0.9933	0.9880	0.9779	0.9584

Using this approximated model for the system, the cumulative distribution of the P_r for different types of users is derived using geometrical area calculations. Firstly we obtain the distribution of the distance (D) of a user to its serving BS. The relation between distance D, transmission power P_t and received power P_r is as given in (2).

$$P_r = \frac{P_t}{D^{\alpha_i}}. \tag{2}$$

Using the distance distributions and (2), the distributions of P_r are obtained as given in Sections II-1, II-2 and II-3.



(a) $B = 0dB$ (b) $B = 20dB$

Fig. 2: Approximated coverage for varying B values

1) Distribution of Received Power for Direct Micro Users: Received power for a UE is a function of the distance (D) between the UE and the BS which is associated with it. Therefore, in order to obtain the received power distribution, first we have to obtain CDF of the random variable D, i.e. $F_D(d) = P(D \leq d)$. As shown in Figure 2a, Direct Micro coverage is approximated by a perfect circle. Using this approximation and keeping uniform user distribution in mind, $F_D(d)$ for a direct micro user can be calculated as

$$P(D \leq d) = \frac{S(d)}{S_{DIR}}. \tag{3}$$

In (3), $S(d)$ is the intersection area of the circle centered at Micro BS location with a radius of d ($d \leq R_{max}$) and approximated coverage region for Micro BS (orange colored region in Figure 3). R_{max} is the maximum distance between micro BS and a Direct Micro UE. S_{DIR} is the area of the approximated direct Micro coverage region (union of orange and green colored regions in Fig. 3). Calculation of the intersection area of two circles is given in III. Using (3) and the relation between distance and received power that is given by (2), the CDF of received power P_r can be expressed as

$$F_{p_r}(P_r) = 1 - F_D(\sqrt[-\alpha_2]{\frac{P_r}{P_2}}). \tag{4}$$

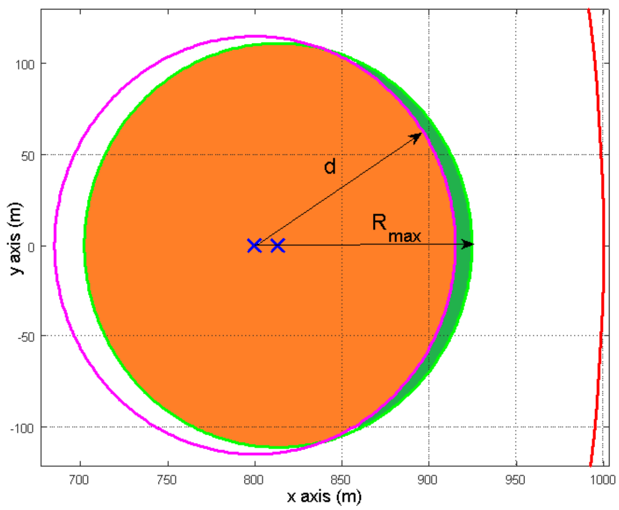


Fig. 3: Calculation of Direct Micro Users' distance distribution to serving Micro BS

2) Distribution of Received Power for CRE UEs: The distribution of P_r for CRE UEs can be calculated similarly to that of Direct Micro UEs. The approximated coverage model for $B = 10dB$ and $B = 20dB$ are shown in Figures 4a and 4b. The range extended coverage of Micro BS is approximated by a perfect circle. Using uniform distribution of UEs, the distribution of the distance between CRE UEs and Micro BS is given by

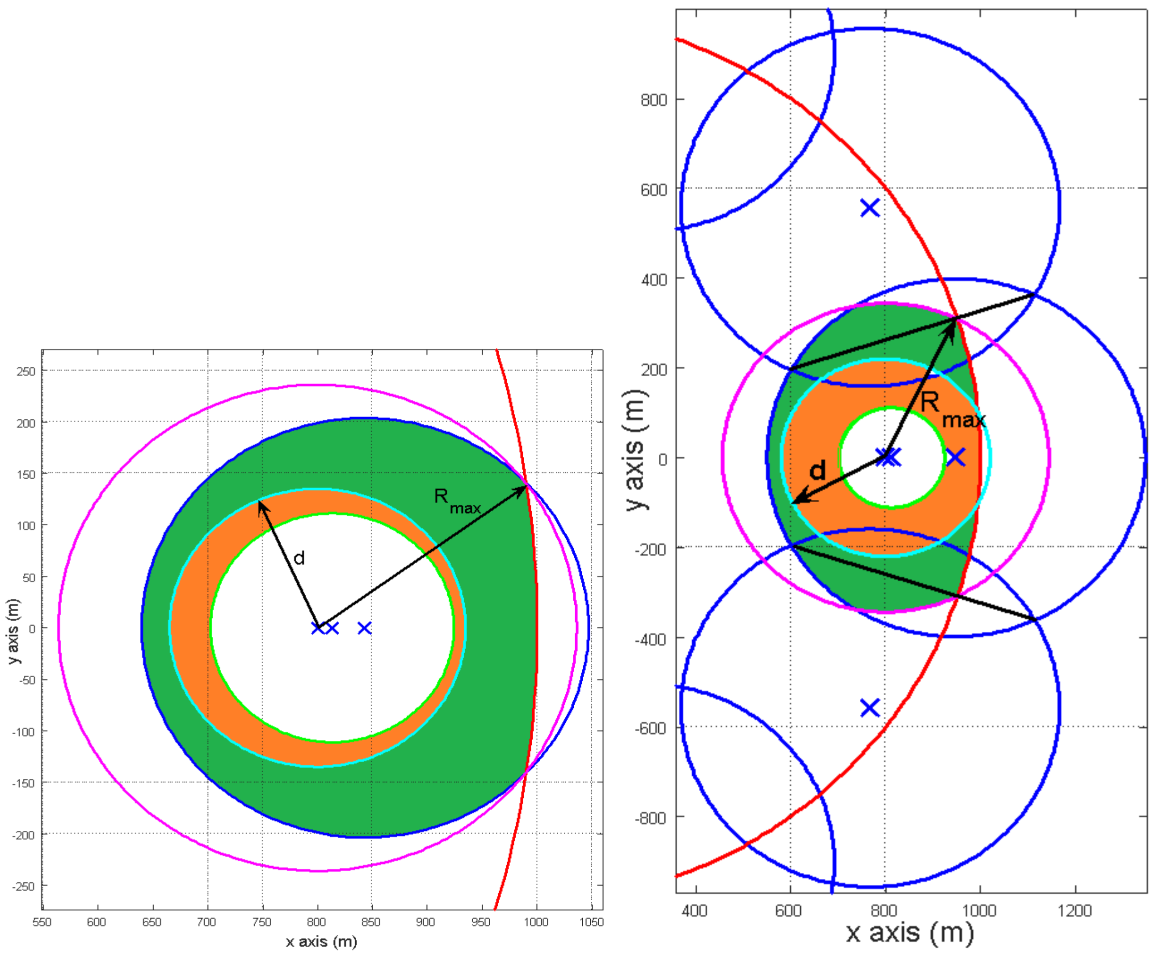
$$P(D \leq d) = \frac{S(d)}{S_{CRE}(B)}. \tag{5}$$

In (5), $S(d)$ is the area of the intersection of the circle centered at Micro BS location with a radius d ($d \leq R_{max}$) and the whole region where UEs are uniformly located. This region is colored as orange in Figures 4a and 4b for different B values. $S_{CRE}(B)$ is the total area of the region where CRE users are located for a given value of B. This area can be expressed by the union of orange and green colored regions that are shown in Figures 4a and 4b. Calculation of the intersection area of three circles is done by using the formulas given in IV.

Using (2) and (5), the CDF of received power, P_r can be written as

$$F_{p_r}(P_r) = 1 - F_D(\sqrt[-\alpha_2]{\frac{P_r}{P_2}}). \tag{6}$$

3) Distribution of Received Power for Macro UEs: Distribution of the P_r for Macro UEs can be found by calculating the distribution of the distance between Macro UEs and Macro BS. The Macro BS coverage region is modeled as a combination of differently shaped regions as



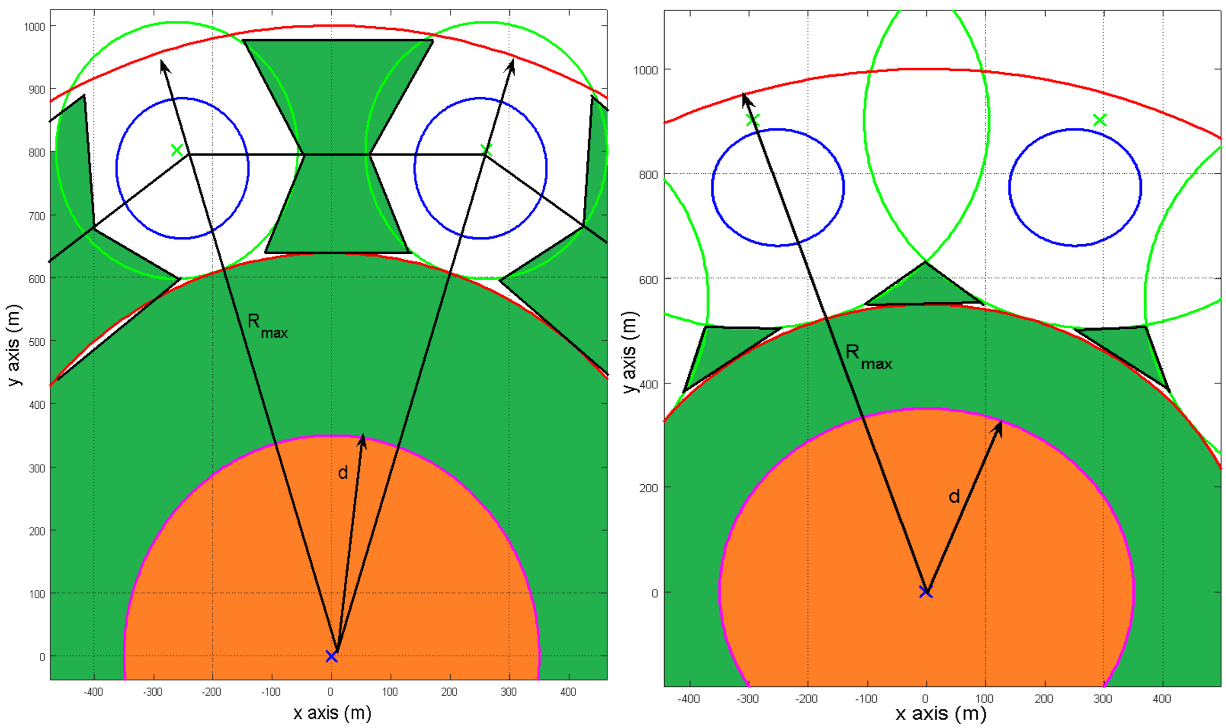
(a) $B = 10dB$ (b) $B = 20dB$

Fig. 4: CRE Region in detail for varying B values

illustrated in Figures 5a and 5b for different B values. The CDF of the distance between Macro UEs and Macro BS is given by

$$P(D \leq d) = \frac{S(d)}{S_{MACRO}(B)}. \tag{7}$$

In (7), $S(d)$ is the area of the intersection of the circle centered at Macro BS location with a radius d ($d \leq R_{max}$), with approximated Macro region. This region is colored to orange in Figures 5a and 5b. $S_{MACRO}(B)$ is the total area of the region where Macro users are uniformly located for a given bias value B, geometrically it is the union of orange and green colored regions in Figures 5a and 5b.



(a) $B = 10dB$

(b) $B = 20dB$

Fig. 5: Macro Region in detail for varying B values

Using (2) and (7), the CDF of received power P_r is given by

$$F_{p_r}(P_r) = 1 - F_D(\sqrt[-\alpha_1]{\frac{P_r}{P_1}}). \tag{8}$$

III. Intersection Area Calculation of Two Circles

Two circles with radius values R_1 and R_2 ($R_1 > R_2$) may be located in four different positions to each other. These cases are shown in Figures 6-9. In this section, formulas for the intersection area of these two circles for these four cases will be derived. The different cases in Figures 6-9 are defined as:

$$Case = \begin{cases} 0, & d > R_1 + R_2 \\ 1, & R_1 < d < R_1 + R_2 \\ 2, & d < R_1 < 2R_2 \\ 3 & d < R_1, R_1 > 2R_2 \end{cases}, \tag{9}$$

In (9), d is the distance between centers of the two circles and is defined by

$$d = |C_1 - C_2|, \tag{10}$$

where C_1 and C_2 are the coordinates of centers of the two circles.

(11) gives the intersection area of two circles for different cases: a and b used in (11) are distances between points C_2, C_3 and C_3, C_4, respectively. Points C_2 , C_3 and C_4 are shown in Figures 7 and 8.

$$S_{intersect} = \begin{cases} 0, & \text{Case 1} \\ cos^{-1}(\frac{a}{R_1})R_1^2 - ab & \\ +cos^{-1}(\frac{d-a}{R_2})R_2^2 - b(d-a), & \text{Case 2, Case 3} \\ \pi R_2^2, & \text{Case 4} \end{cases} \tag{11}$$

where

$$a = \frac{-R_2^2 + R_1^2 + d^2}{2d}, \tag{12}$$

$$b = \sqrt{R_1^2 - a^2}, \tag{13}$$

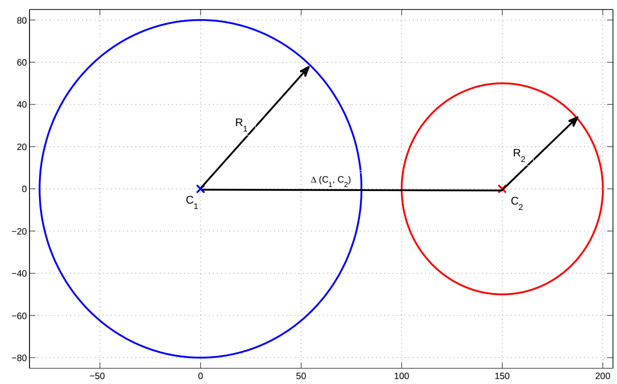


Fig. 6: Intersection of two circles, Case-1

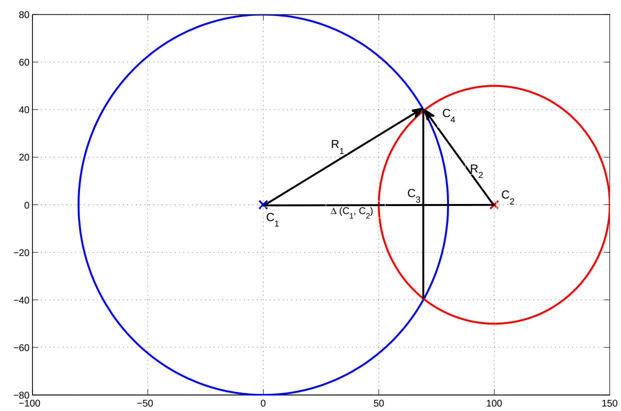


Fig. 7: Intersection of two circles, Case-2

IV. Intersection Area Calculation of Three Circles

In order to properly calculate the region where CRE UEs are located, intersection area of three circles should be calculated. According to our system model, the red colored area in Fig. 10 should be obtained. Using definitions shown in Fig. 10, the red area $S_{Int,3}$ can be expressed as

$$S_{Int,3} = 2(S_1 + S_2) \tag{14}$$

where

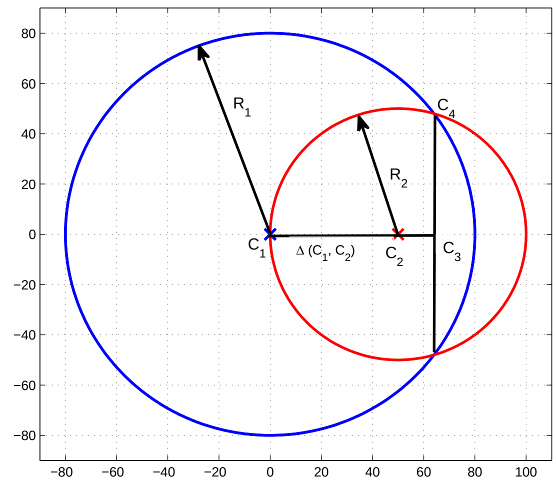


Fig. 8: Intersection of two circles, Case-3

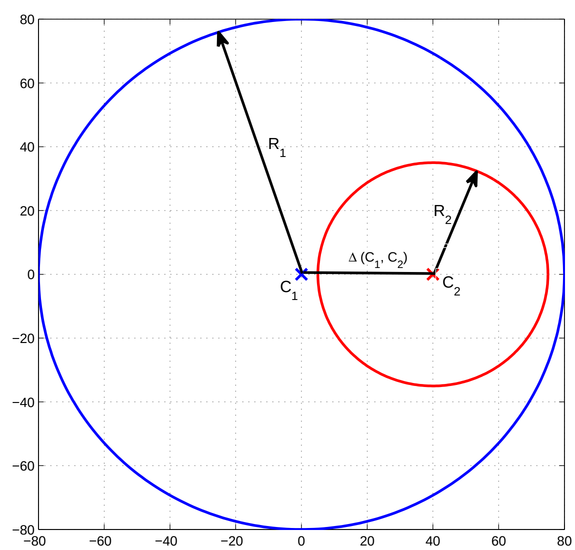


Fig. 9: Intersection of two circles, Case-4

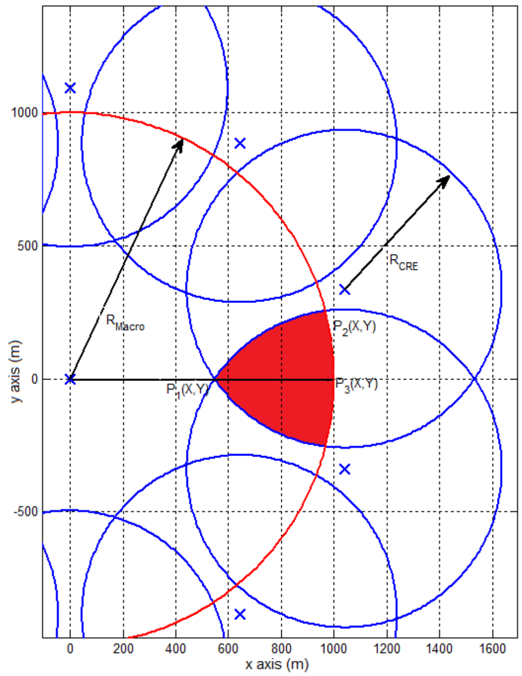


Fig. 10: Intersection Area of 3 Circles

$$S_1 = \int_{P_{1,x}}^{P_{2,x}} \sqrt{-(x - C_{1,x})^2 + R_{CRE}^2} + C_{1,y} dx, \tag{15}$$

$$S_2 = \int_{P_{2,x}}^{P_{3,x}} \sqrt{-(x - C_{2,x})^2 + R_{Macro}^2} + C_{2,y} dx \tag{16}$$

In (15), (16), $P_{1,x}$, $P_{2,x}$, $P_{3,x}$ are the x coordinates of the points $P_1(X,Y)$, $P_2(X,Y)$ and $P_3(X,Y)$, respectively. $P_1(X,Y)$ is the point where two adjacent CRE circle intersect, $P_2(X,Y)$ is the point where Macro circle and CRE circle intersects, $P_3(X,Y)$ is the point where x axes and Macro circle intersects. $C_{1,x}$, $C_{1,y}$, $C_{2,x}$ and $C_{2,y}$ are the x and y coordinates of the centers of Macro and CRE circles respectively. R_{Macro} and R_{CRE} are the radius values of Macro and CRE circles. All these points and variables are shown in Figure 10.

References

[1] G. Yenihayat and E. Karaşan, "Downlink data rate, energy and spectral efficiency distribution in heterogeneous networks with cell-edge located small cells," *Submitted to Wireless Networks*, 2018.