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Energetic efficient synthesis of general mutual intensity distribution

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Abstract. The mutual intensity distribution of a light beam may contain available information. The task of encoding a given mutual intensity distribution is addressed in this paper. Various approaches for encoding the mutual intensity function have been previously proposed. However, all of them provide low energetic efficiency and commonly require sophisticated production methods. The idea of using a phase-only filter for performing this synthesis is hereby investigated. The proposed method is numerically examined for the case of placing the mutual intensity generating filter in the fractional Fourier domain.

Keywords: Statistical optics, mutual intensity, fractional Fourier transform, phase-only filtering

1. Introduction

The behaviour of optical systems is influenced by the degree of partial coherence. In this paper a quasi-monochromatic light with different degrees of spatial coherence is considered. Note that the term 'spatially coherent' implies a deterministic relation between two points of the scene.

A detailed treatment of spatially incoherent systems is well introduced by [1, 2]. The mutual intensity of a scalar field distribution $u(P, t)$ is defined as

$$J_{12}(P_1, P_2) = \langle u(P_1, t)u^*(P_2, t)\rangle \tag{1}$$

where $\langle\cdots\rangle$ is a time-averaging operation. The normalized mutual intensity becomes

$$\mu_{12} = \frac{J_{12}(P_1, P_2)}{\sqrt{I(P_1)I(P_2)}} \tag{2}$$

where $I(P)$ is the intensity at location P.

These definitions are adequate for a quasi-monochromatic beam. For the more general case of a spatially and temporally incoherent beam, one needs to use the mutual coherence function defined as

$$\Gamma_{12}(\tau) = \langle u(P_1, t+\tau)u^*(P_2, t)\rangle. \tag{3}$$

Its normalized value is denoted as $\gamma_{12}(\tau)$. Obviously, $\Gamma_{12}(0) = J_{12}(P_1, P_2)$.

For totally temporally and spatially coherent light one has

$$|\gamma_{12}(\tau)| = |\mu_{12}| = 1 \tag{4}$$

while for spatially incoherent light one has

$$J_{12}(P_1, P_2) = kI(P_1)\delta(P_1 - P_2) \tag{5}$$

and

$$|\gamma_{12}(\tau)| = |\mu_{12}| = \delta(P_1 - P_2). \tag{6}$$

In [3], the propagation of mutual intensity through linear quadratic-phase systems, having a transformation kernel of B_p (where p is the transformation order), was shown to be expressed as

$$J_p(x_1, x_2) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} B_p(x_1, x_1') \times B_{-p}(x_2, x_2')J(x_1', x_2')\,\mathrm{d}x_1'\,\mathrm{d}x_2' \tag{7}$$

where J is the given mutual intensity and J_p is the mutual intensity of the light after propagating through this system. Note that the fractional order p is related to the free space propagation distances and the focal lengths of the lenses used in the optical implementation of the fractional Fourier transformer. The authors of [3] discussed the case where the transformation used is the fractional Fourier transform (FRT) [4, 5]. Thus, in that case J_p is a two-dimensional FRT over the given mutual intensity J. In [6], this property was used to synthesize (by proper filtering in the FRT domain) a desired mutual intensity distribution. Additional recent works dealing with mutual intensity synthesis may be found in [7–10].

In this paper, we develop a mathematical expression for synthesizing a desired mutual intensity distribution by using a phase-only filter. This way the obtained light efficiency is much higher and the production process of the filter's mask is much simpler. The computer simulations demonstrate the synthesis ability using the phase-only filter for a transformation kernel B_p which is the kernel of the FRT. However, the presented mathematical analysis is completely general and valid for any other kernel of transformation as well. For instance, a more general case is when the chosen kernel B_p is the kernel of a canonical $ABCD$

transformation [11, 12]. Note that when the FRT kernel is used a proper optimization in a sense of minimal mean-square error is applied to find the optimal fractional order p of the transformation. As previously mentioned, variation of the fractional order is related to changes in the free space distances and the focal lengths of the lenses used in the optical setup which implements the fractional Fourier transformer module.

2. Motivation

The importance of the mutual coherence function is revealed when considering a partially coherent system, in which the signal cannot be represented properly by the complex amplitude or the intensity distributions due to its 'random-process' behaviour.

The initial motivation for this work was inspired by the well known interview which Gabor gave when his Nobel Prize was announced. When asked the question, 'When did you start working on holography?' Gabor replied, 'As a high-school student I was puzzled that a photographic plate midway between object and camera would show no structure or pattern after being exposed and developed. Yet, the information of the object must travel through that plane on its way to the camera. How does the light carry information from the object to the proper image plane?'

The answer to that question is fairly simple when considering the coherence properties of light waves, for the information travels by means of the mutual coherence function. Had we measured the coherence function at any arbitrary plane midway between the object and the camera, all the information could have been extracted.

One of the conclusions from the above discussion is that the mutual coherence function is a carrier of information in optical systems, but is not utilized as such. The main reason is that it is relatively easy to encode and decode information from intensity or complex amplitude distributions by use of SLMs and CCDs, while synthesizing and analysing the coherence function of a light beam is much more difficult. Coherent optical systems for signal processing are noisy and exhibit speckles at their output. Evidently, incoherent signal processing, which relies upon the intensity distribution as a carrier of information, overcomes the speckle effect, but usually requires more energy and allows only real and non-negative inputs, i.e., less degrees of freedom. In order to overcome the speckle noise and at the same time allow complex amplitude distributions, a different type of processing is required. Thus, having the ability to synthesize and process mutual coherence functions with high energetic efficiency gains ultimate significance. For instance, once measuring the mutual coherence function the distortion parameters of the media may be extracted. Synthesizing the optimal distribution may allow us to overcome these distortions; constructing a coherence processor allows one to implement a more discriminate and more accurate optical data processing device. A synthesis of special coherence functions may be very applicable in the construction of accurate measuring instrumentation (such as distance measurement based on the coherence length or profile meter).

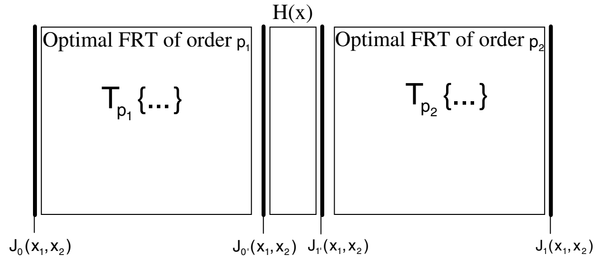

Figure 1. Schematic illustration of the mutual intensity propagated through a fractional Fourier transforming system.

3. Mathematical analysis

The filtering setup is schematically illustrated in figure 1. In this figure the transformation kernel is the kernel of the FRT. We denote by J_1^d the desired output mutual intensity, by J_1 the obtained output mutual intensity, and by J_0 the input mutual intensity. The definition of the mean-square error is

$$M = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} |J_1^d(x_1, x_2) - kJ_1(x_1, x_2)|^2 \, dx_1 \, dx_2 \qquad (8)$$

where k is a constant. This constant does not affect the resulting phase-only filter and thus we can omit it from the mathematical analysis. For unitary transformations (such as the FRT), M may also be expressed on plane $1'$ of figure 1:

$$M = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} |J_{1'}^d(x_1, x_2) - kJ_{1'}(x_1, x_2)|^2 \, dx_1 \, dx_2. \qquad (9)$$

Referring again to figure 1, one may note that

$$\begin{aligned} J_{1'}(x_1, x_2) &= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} B_{-p_2}(x_1, x_1') \\ &\quad \times B_{p_2}(x_2, x_2') J_1(x_1', x_2') \, dx_1' \, dx_2' \\ J_{0'}(x_1, x_2) &= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} B_{p_1}(x_1, x_1') \\ &\quad \times B_{-p_1}(x_2, x_2') J_0(x_1', x_2') \, dx_1' \, dx_2' \end{aligned} \qquad (10)$$

and that the input–output spectral mutual intensities relationship is

$$J_{1'}(x_1, x_2) = J_{0'}(x_1, x_2) H(x_1) H^*(x_2) \qquad (11)$$

where H is our phase-only filter placed in the transformation domain:

$$H(x) = \exp[iw(x)] \qquad (12)$$

and w is the phase of the filter. Thus, using equations (9), (11) and (12) one obtains

$$\begin{aligned} M = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} &\{|J_{1'}^d(x_1, x_2)|^2 + |H(x_1)H^*(x_2)J_{0'}(x_1, x_2)|^2 \\ &- 2\mathrm{Re}\,[J_{1'}^{d^*}(x_1, x_2) H(x_1) H^*(x_2) J_{0'}(x_1, x_2)]\} \, dx_1 \, dx_2. \end{aligned} \qquad (13)$$

Note that since we are dealing with a phase-only filter

$$|H(x_1)H^*(x_2)|^2 = 1 \qquad (14)$$

and thus

$$\begin{aligned} M = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} &\{|J_{1'}^d(x_1, x_2)|^2 + |J_{0'}(x_1, x_2)|^2 \\ &- 2\mathrm{Re}\,[J_{1'}^{d^*}(x_1, x_2) H(x_1) H^*(x_2) J_{0'}(x_1, x_2)]\} \, dx_1 \, dx_2. \end{aligned} \qquad (15)$$

Now we will allow variations in the phase of the filter

$$w(x) \longrightarrow w(x) + \delta_w(x) \qquad (16)$$

and observe their effect over M. The optimal phase-only filter is the filter whose variations cause zero effects over M because only the optimal filter minimizes M. The condition that imposes zero effects over M, will yield an iterating equation for the phase-only filter as we are about to derive. In order to avoid convergence to local minimas, the initial conditions of the iterative equation have to be wisely chosen. We will assume that the variations are small and that terms such as δ_w^2 are negligible:

$$\exp\{\mathrm{i}[w(x) + \delta_w(x)]\} \approx \exp[\mathrm{i}w(x)][1 + \mathrm{i}\delta_w(x)]. \qquad (17)$$

Substituting this relation in equation (15), yields

$$\begin{aligned} M^\delta = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} &\{|J_{1'}^d(x_1, x_2)|^2 + |J_{0'}(x_1, x_2)|^2 \\ &- 2\mathrm{Re}\,[J_{1'}^{d^*}(x_1, x_2) J_{0'}(x_1, x_2) \exp[\mathrm{i}(w(x_1) \\ &- w(x_2))][1 + \mathrm{i}(\delta_w(x_1) - \delta_w(x_2))]]\}\,\mathrm{d}x_1\,\mathrm{d}x_2. \end{aligned} \qquad (18)$$

We will define

$$M^\delta = M + \epsilon_M \qquad (19)$$

where M is the part that does not depend upon the phase variations of the filter. From equation (18) one obtains

$$\begin{aligned} \epsilon_M = 2\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} &\{\mathrm{Re}\,[J_{1'}^{d^*}(x_1, x_2) J_{0'}(x_1, x_2)][\delta_w(x_1) \\ &- \delta_w(x_2)] \sin(w(x_1) - w(x_2)) \\ &+ \mathrm{Im}\,[J_{1'}^{d^*}(x_1, x_2) J_{0'}(x_1, x_2)][\delta_w(x_1) \\ &- \delta_w(x_2)] \cos(w(x_1) - w(x_2))\}\,\mathrm{d}x_1\,\mathrm{d}x_2. \end{aligned} \qquad (20)$$

Rewriting the last equation, we have

$$\begin{aligned} \epsilon_M = 2\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} &\{\delta_w(x_1)[\mathrm{Re}\,[J_{1'}^{d^*}(x_1, x_2) \\ &\times J_{0'}(x_1, x_2)] \sin(w(x_1) - w(x_2)) \\ &+ \mathrm{Im}\,[J_{1'}^{d^*}(x_1, x_2) J_{0'}(x_1, x_2)] \cos(w(x_1) - w(x_2))] \\ &- \delta_w(x_2)[\mathrm{Re}\,[J_{1'}^{d^*}(x_1, x_2) \\ &\times J_{0'}(x_1, x_2)] \sin(w(x_1) - w(x_2)) \\ &+ \mathrm{Im}\,[J_{1'}^{d^*}(x_1, x_2) J_{0'}(x_1, x_2)] \\ &\times \cos(w(x_1) - w(x_2))]\}\,\mathrm{d}x_1\,\mathrm{d}x_2. \end{aligned} \qquad (21)$$

Performing a change of variables $x_2 \longrightarrow x_1$ and $x_1 \longrightarrow x_2$ in the first term of equation (21), yields

$$\begin{aligned} \epsilon_M = 2\int_{-\infty}^{\infty} \delta_w(x_2) \bigg\{ \int_{-\infty}^{\infty} &-\mathrm{Re}\,[J_{1'}^{d^*}(x_2, x_1) J_{0'}(x_2, x_1) \\ &+ J_{1'}^{d^*}(x_1, x_2) J_{0'}(x_1, x_2)] \times \sin(w(x_1) - w(x_2)) \\ &+ \mathrm{Im}\,[J_{1'}^{d^*}(x_2, x_1) J_{0'}(x_2, x_1) - J_{1'}^{d^*}(x_1, x_2) J_{0'}(x_1, x_2)] \\ &\times \cos(w(x_1) - w(x_2))\,\mathrm{d}x_1 \bigg\} \mathrm{d}x_2. \end{aligned} \qquad (22)$$

Since we wish ϵ_M to be zero for any possible phase variation δ_w, we restrict

$$\begin{aligned} \int_{-\infty}^{\infty} &\{G_R(x_1, x_2) \sin(w(x_1) - w(x_2)) \\ &+ G_I(x_1, x_2) \cos(w(x_1) - w(x_2))\}\,\mathrm{d}x_1 = 0 \end{aligned} \qquad (23)$$

where

$$\begin{aligned} G_R(x_1, x_2) &= -\mathrm{Re}\,[J_{1'}^{d^*}(x_2, x_1) J_{0'}(x_2, x_1) \\ &\quad + J_{1'}^{d^*}(x_1, x_2) J_{0'}(x_1, x_2)] \\ G_I(x_1, x_2) &= \mathrm{Im}\,[J_{1'}^{d^*}(x_2, x_1) J_{0'}(x_2, x_1) \\ &\quad - J_{1'}^{d^*}(x_1, x_2) J_{0'}(x_1, x_2)]. \end{aligned} \qquad (24)$$

Using the trigonometric relations

$$\begin{aligned} \sin(w(x_1) - w(x_2)) &= \sin(w(x_1)) \cos(w(x_2)) \\ &\quad - \cos(w(x_1)) \sin(w(x_2)) \\ \cos(w(x_1) - w(x_2)) &= \sin(w(x_1)) \sin(w(x_2)) \\ &\quad + \cos(w(x_1)) \cos(w(x_2)) \end{aligned} \qquad (25)$$

one obtains

$$\begin{aligned} \sin(w(x_2))\bigg[\int_{-\infty}^{\infty} &(G_R(x_1, x_2) \cos(w(x_1)) \\ &+ G_I(x_1, x_2) \sin(w(x_1)))\,\mathrm{d}x_1 \bigg] \\ = \cos(w(x_2))\bigg[\int_{-\infty}^{\infty} &(G_R(x_1, x_2) \sin(w(x_1)) \\ &- G_I(x_1, x_2) \cos(w(x_1)))\,\mathrm{d}x_1 \bigg]. \end{aligned} \qquad (26)$$

Thus,

$$\begin{aligned} \tan(w(x_2)) = \bigg\{ \int_{-\infty}^{\infty} &(G_R(x_1, x_2) \sin(w(x_1)) \\ &- G_I(x_1, x_2) \cos(w(x_1)))\,\mathrm{d}x_1 \bigg\} \\ \times \bigg\{ \int_{-\infty}^{\infty} &(G_R(x_1, x_2) \cos(w(x_1)) \\ &+ G_I(x_1, x_2) \sin(w(x_1)))\,\mathrm{d}x_1 \bigg\}^{-1}. \end{aligned} \qquad (27)$$

Note that equation (27) is an iterating equation, from assuming a certain solution for $w(x_1)$, one may compute the phase $w(x_2)$ in the next iteration:

$$\begin{aligned} w(x_2) = \arctan \bigg[\bigg\{ \int_{-\infty}^{\infty} &(G_R(x_1, x_2) \sin(w(x_1)) \\ &- G_I(x_1, x_2) \cos(w(x_1)))\,\mathrm{d}x_1 \bigg\} \\ \times \bigg\{ \int_{-\infty}^{\infty} &(G_R(x_1, x_2) \cos(w(x_1)) \\ &+ G_I(x_1, x_2) \sin(w(x_1)))\,\mathrm{d}x_1 \bigg\}^{-1} \bigg]. \end{aligned} \qquad (28)$$

According to equation (28) the phase of the optimal filter depends upon the fractional orders p_1 and p_2 of the transformation kernel. We return again to the expression for M in equation (8). This expression may also be minimized as a function of p_1 and p_2. Thus, the optimal filter should be found for various values of p_1 and p_2: two vectors $\vec{p}_1$ and $\vec{p}_2$ should be defined. Each vector contains all possible values for the fractional orders:

$$\vec{p}_1 = \vec{p}_2 = [\,-2 \quad -2 + \delta p \quad -2 + 2\delta p \quad \cdots \quad 2 - \delta p \quad 2\,] \qquad (29)$$

where δp is the fractional order computation resolution. For each component in the $\vec{p}_1$, $\vec{p}_2$ vectors a mean-square error,

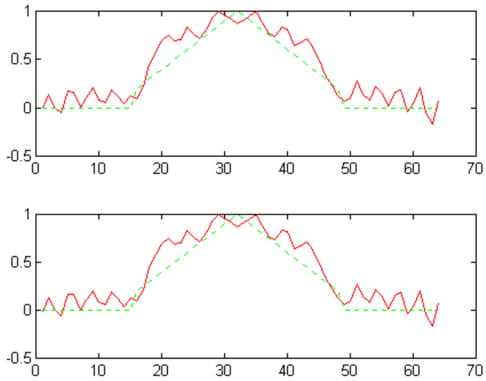

Figure 2. The horizontal and the vertical (upper and lower plots, respectively) cross sections of the mutual intensity distribution. Solid curve: obtained output mutual intensity distribution. Dashed curve: the desired mutual intensity distribution.

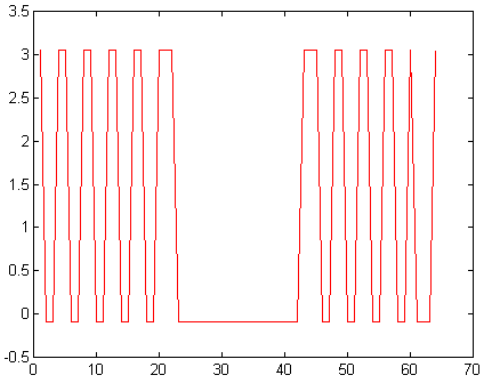

Figure 3. The phase of the phase-only filter.

$M(p_1, p_2)$, between the desired and the obtained mutual intensities, should be computed. The pair of values p_1, p_2 for which the matrix $M(p_1, p_2)$ achieves the minimal value is chosen to be the optimal fractional order pair.

Note that equation (28) is a general equation and thus the same procedure may be applied for a general $ABCD$ kernel.

4. Computer simulation

In order to examine the ability of the suggested approach, computer simulations were performed. A given mutual intensity, similar to the one assumed in [6], was taken; specifically

$$J_0(x_1, x_2) = \exp\left(-\frac{(x_1 - x_2)^2}{2\sigma^2}\right), \tag{30}$$

which means that the correlation between each two spatial points in the illuminating source behaves as Gaussian function of the difference between the points. σ is the width of the Gaussian. Note that for $\sigma \to 0$, J_0 becomes spatially incoherent light $J_0(x_1, x_2) = \delta(x_1 - x_2)$. For $\sigma \to \infty$ the light becomes fully coherent since J_0 is then a constant. In the simulations we took $\sigma = 14.31$ pixels.

To examine the ability of the suggested approach one may choose a non-physical desired distribution and then the obtained result will be the closest (in the sense of minimal

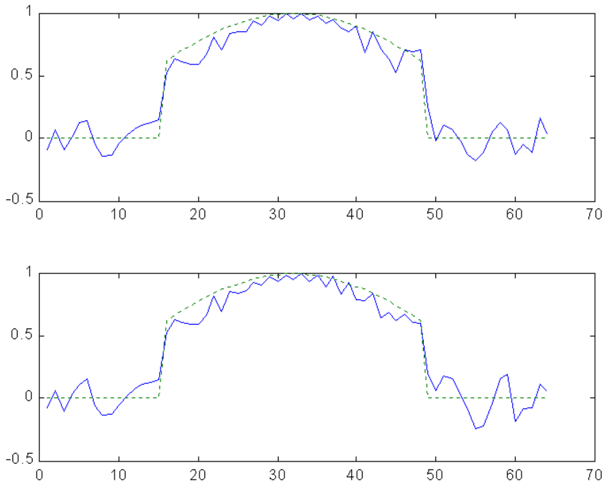

Figure 4. The horizontal and the vertical (upper and lower plots, respectively) cross sections of the mutual intensity distribution. Solid curve: obtained output mutual intensity distribution. Dashed curve: the desired mutual intensity distribution.

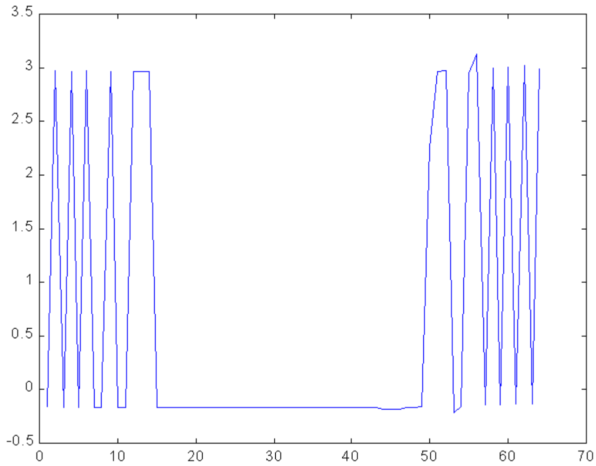

Figure 5. The phase of the phase-only filter.

mean-square error) physical distribution. However, in such a case part of the remaining convergence error is due to the non-feasibility of the solution. Thus, we choose to examine the suggested technique by converging it to several physically feasible and commonly used [1] desired distributions. In the first simulation we chose the following desired distribution:

$$J_1^d(x_1, x_2) = \Lambda\left(\frac{|x_1 - x_2|}{2r_1}\right) \operatorname{rect}\left(\frac{x_1}{2r_2}\right) \operatorname{rect}\left(\frac{x_2}{2r_2}\right) \tag{31}$$

with $r_1 = 10$, $r_2 = 17$ pixels. Λ is a triangle function and rect is a rectangle function:

$$\Lambda\left(\frac{x}{\Delta x}\right) = 1 - \left|\frac{x}{\Delta x}\right| \tag{32}$$

for $|x| < \Delta x$ and zero otherwise, and

$$\operatorname{rect}\left(\frac{x}{\Delta x}\right) = 1 \tag{33}$$

for $|x| < \Delta x/2$ and zero otherwise.

The iterating formula of equation (28) was used in order to obtain the desired synthesis while the transformation kernel was chosen to be the kernel of the FRT. The iterating algorithm was investigated for different fractional orders. The best synthesis was obtained for $p_1 = 0.71$, $p_2 = -0.71$.

The solid curve shown in figure 2 illustrates the cross section of the obtained 2D output mutual intensity function while the dashed curve illustrates the cross section of the desired 2D mutual intensity function. The upper part of the figure is a horizontal and the lower part is a vertical cross section of the 2D mutual intensity function. Figure 3 illustrates the obtained phase of the computed phase-only filter.

Another simulation was performed with a desired distribution of

$$J_1^d(x_1, x_2) = \operatorname{sinc}\left(\frac{x_1 - x_2}{r_1}\right) \operatorname{rect}\left(\frac{x_1}{2r_2}\right) \operatorname{rect}\left(\frac{x_2}{2r_2}\right) \quad (34)$$

once again with $r_1 = 10$, $r_2 = 17$ pixels, while the sinc function is defined as

$$\operatorname{sinc}(x) = \frac{\sin x}{x}. \quad (35)$$

In this case the best synthesis was obtained for $p_1 = 0.27$, $p_2 = -0.27$ while here once again the transformation kernel was chosen to be the kernel of the FRT.

The solid curve shown in figure 4 illustrates the cross section of the obtained 2D output mutual intensity function while the dashed curve illustrates the cross section of the desired 2D mutual intensity function. The upper part of the figure is a horizontal and the lower part is a vertical cross section of the 2D mutual intensity function. Figure 5 illustrates the obtained phase of the computed phase-only filter.

5. Conclusions

In this paper a new iterating algorithm was derived for obtaining the phase-only filter which is optimal for synthesizing a desired mutual intensity distribution. The synthesis was achieved by placing this phase-only filter in the transformation domain, as illustrated in figure 1. The computer simulations demonstrated the synthesis ability for a FRT kernel. However, the mathematical derivation is completely general and could be used for any other transformation's kernel. Except for the iteration procedure, the chosen fractional orders of the filtering system were found to optimize the synthesized distribution (they minimized the mean-square error).

Since the obtained filter was a phase-only filter, high efficiency was achieved in the output plane. In addition, the production process of a phase-only filter is much simpler than producing a general filter containing both phase and amplitude.

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