SINGLE DIPOLE THEORY OF THE ELECTROCARDIOGRAM (ECG):  
EINDHOVEN’S TRIANGLE

Let us assume that the bioelectric sources in the heart can be expressed as a single equivalent dipole current source the strength and direction of which change in time during the PQRST complex. We further assume that the heart is at the center of a spherical conducting volume of radius R, which represents the torso. We wish to find the distribution of electrical potential on the surface of the torso. For a bounded space like the sphere representing the torso we must solve a boundary value problem.

Let us first assume that we have a dipole \( \vec{P} = P\vec{u}_y \) at the origin as shown below.

Except at the origin where the heart dipole exists \( \nabla^2 \Phi = 0 \), and on the surface \( \frac{\partial \Phi}{\partial r} \bigg|_{r=R} = 0 \) which means the electrical field and also the current density cannot be normal to the surface; they must be parallel to it. In general \( \Phi = \sum_{n=0}^{\infty} \left[ a_n r^n P_n(\cos \theta) + b_n r^{-(n+1)} P_n(\cos \theta) \right] \) where \( P_n \) are legendre polynomials.

Since near the origin \( \Phi \) must vary as \( \frac{1}{r^2} \) we take \( n = 1 \), we can only take the \( n = 1 \) term.

Therefore \( \Phi = a_r \cos \theta + b_r r^{-2} \cos \theta \) (because \( P_1(\cos \theta) = \cos \theta \)). Since

\[
\frac{\partial \Phi}{\partial r} \bigg|_{r=R} = a_i \cos \theta - 2b_i R^{-3} \cos \theta \bigg|_{r=R} = a_i \cos \theta - 2b_i R^{-3} \cos \theta = 0 \quad \text{we have} \quad a_i = \frac{2b_i}{R^3}.
\]

Finally \( \Phi(r) = \frac{b_i}{R^3} \left[ \frac{2r}{R} + \left( \frac{R}{r} \right)^2 \right] \cos \theta \), and the potential distribution on the surface

\[
\Phi(R) = \frac{3b_i}{R^2} \cos \theta.
\]

Next consider a heart dipole at the origin which makes an angle \( \alpha \) with the horizontal axis.
Measurement locations are the left arm (LA), right arm (RA), and the left leg (LL) which are located at the angles $30^\circ$, $150^\circ$ and $-90^\circ$ with the horizontal axis, respectively.

Using the previous solution, we can say that, apart from a constant multiplying factor,

$$
\Phi_{LA} = \cos(\alpha - 30^\circ) = \frac{\sqrt{3}}{2} \cos \alpha + \frac{1}{2} \sin \alpha
$$

$$
\Phi_{RA} = \cos(150^\circ - \alpha) = -\frac{\sqrt{3}}{2} \cos \alpha + \frac{1}{2} \sin \alpha
$$

$$
\Phi_{LL} = \cos(\alpha + 90^\circ) = -\sin \alpha
$$

We define the standard lead measurement voltages $V_1$, $V_2$, and $V_3$ and using the equalities

$$
\cos(x \pm y) = \cos x \cos y \mp \sin x \sin y
$$

$$
\sin(x \pm y) = \sin x \cos y \pm \cos x \sin y
$$

we get

$$
V_1 \doteq \Phi_{LA} - \Phi_{RA} = \sqrt{3} \cos \alpha
$$

$$
V_2 \doteq \Phi_{LL} - \Phi_{RA} = \sqrt{3} \left(\frac{1}{2} \cos \alpha - \frac{\sqrt{3}}{2} \sin \alpha\right)
$$

$$
= \sqrt{3} \left(\cos 60^\circ \cos \alpha - \sin 60^\circ \sin \alpha\right)
$$

$$
= \sqrt{3} \cos(\alpha + 60^\circ)
$$

$$
V_3 \doteq \Phi_{LL} - \Phi_{LA} = \sqrt{3} \left(-\frac{1}{2} \cos \alpha - \frac{\sqrt{3}}{2} \sin \alpha\right)
$$

$$
= \sqrt{3} \left(-\cos 60^\circ \cos \alpha - \sin 60^\circ \sin \alpha\right) = -\sqrt{3} \cos(60^\circ - \alpha)
$$

$$
= \sqrt{3} \cos(\alpha + 120^\circ)
$$
The triangle above is called the Eindhoven’s Triangle. From the expressions we have obtained for $V_1$, $V_II$, and $V_{III}$ we see that apart from a constant factor, $V_1$ is the projection of the dipole onto the line from RA to LA, $V_II$ is the projection of the dipole onto the line from RA to LL, and $V_{III}$ is the projection of the dipole onto the line from LA to LL. In other words $V_1$ is the projection of the dipole onto the lead vector I, $V_II$ is the projection of the dipole onto the lead vector II, and $V_{III}$ is the projection of the dipole onto the lead vector III. Lead vectors are shown in the figure. A lead vector is defined as follows: If we measure the potential of point1 minus the the potential of point2 then the lead vector is defined from point2 to point1. Note that in the above figure the heart vector is such that $V_1$ is positive but $V_II$ and $V_{III}$ are negative.

$V_1$, $V_II$, and $V_{III}$ are not independent and from Kirchoff’s Voltage Law (KVL) $V_1 + V_{III} = V_II$.

If we measure any two of $V_1$, $V_II$, and $V_{III}$ we can calculate the third.

Also using Eindhoven’s Triangle we can find the relative strength of the heart dipole and its direction using only any two of $V_1$, $V_II$, and $V_{III}$ by backprojection.

An interesting observation is made: Suppose we measure for example $V_1$ and $V_II$, and calculate $V_{III}$ using i) KVL, and ii) Einhoven’s Triangle. It is observed that the two calculations match fairly well indicating that the single dipole theory of the heart, that is the theory of Eindhoven, is fairly accurate despite the fact that we have made many assumptions about the heart, namely that, i) it is a single dipole, ii) the torso is a sphere, iii) heart is at the center of the sphere, and iv) measurement positions are at the angles assumed above.

The lead voltages $V_1$, $V_II$, and $V_{III}$ change with time as the heart undergoes the sequence of electrical activity which accompanies its contraction. Thus if we draw the tip of the heart dipole as a function of time we obtain the so called Vectorcardiogram (VCG). On the average the heart vector has a negative angle ($\alpha_{av} < 0$).
In general in 3 dimensions if the heart vector is $\vec{P} = P_x \vec{u}_x + P_y \vec{u}_y + P_z \vec{u}_z$, then the potential difference between two points on the body will be $V = C_x P_x + C_y P_y + C_z P_z = \vec{P} \cdot \vec{C}$ where $\vec{C} = C_x \vec{u}_x + C_y \vec{u}_y + C_z \vec{u}_z$ is the coefficient vector characterizing the measurement positions. In other words $\vec{C}$ is the lead vector for that measurement.