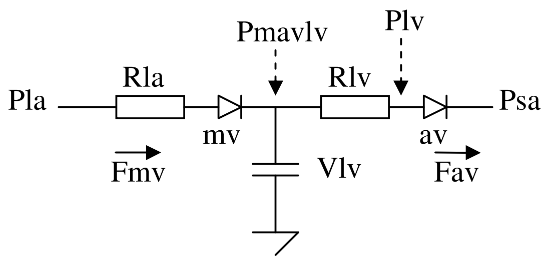
Model of the ventricle

1. Model of the ventricle developed by Ursino

Pla : Left atrial pressure
MV: Mitral valve
Rla: resistance of MV
AV: Aortic valve
Vlv: Volume of left ventricle
Psa: Aortic pressure
Plv: Venricular pressure
Pmaxlv: Isometric pressure that can be developed in the left ventricle
Fav = Flow through the aortic valve
Fmv = Flow through the mitral valve



Although Pla and Psa also change during the cardiac cycle, for the purpose of studying the behavior of the vetricle only, we can assume them to be constant at Pla = 7 mmHg and Psa = 95 mmHg.
Rla = 2.5e-3 mmHg.s/ml
In the above model aortic valve resistance is taken as zero.
Rlv is the viscous resistance of the ventricular wall. Thus if Vlv is not changing (isometric condition) then Plv = Pmaxlv, but if the ventricle is emptying then Plv = Pmaxlv – Rlv*Fav. The viscous resistance is not constant however and changes with Pmaxlv as the cardiac muscle contracts: $Rlv = Krlv \times P\max lv \quad where \quad Krlv = 3.75\times10^{-4}\, s/ml$.
Pmaxlv is dependent on the ventricular volume as well as on time, and is given by

$$P\max lv = \phi(t)\times Elv\max\times(Vlv - Vulv) + (1-\phi(t))\times Polv\times(e^{Kelv\times Vlv} - 1)$$

where

$$Elv\max = 2.95\,mmHg/ml$$

$$Vulv = 16.77\,ml$$

$$Polv = 1.5\,mmHg$$

$$Kelv = 0.014\,ml^{-1}$$

$$\phi(t) = \sin^2\left(\frac{2\pi}{2Tsys}t\right) \quad for \quad 0 \le t \le Tsys$$

$$= 0 \quad for \quad Tsys < t \le T$$

$$and\ repeats\ after\ T$$

where Tsys is systole time and T is the total cardiac period (in seconds). Tsys decreases with increasing heart rate as

$$Tsys = Tsyso - \frac{ksys}{T} \;\; with \; Tsyso = 0.5s \; and \;\; ksys = 0.075s^2 .$$

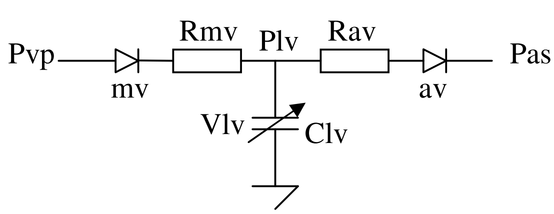
$\phi(t)$ represents the activation of the heart muscle and thus models its variable compliance. In fact the first term in the expression of $\phi(t)$ is a linear compliance term where $\phi(t)\times Elv\max$ is one over compliance and $Vulv$ is the unstressed volume of the ventricle. $Elv\max$ represents the maximum possible elestance (elactance = 1/compliance).The second term basically represents the passive non-linear elastance property of the ventricular mass during diastole. During systole the first term is dominant and during diastole the second term is dominant.

The parameters given avove are for a 70 Kg healthy man and are extracted from human experiments where possible and are extrapolated from dog experiments where needed.

Pla is the called the preload of the ventricle and Psa is called the afterload of the ventricle. If Pla is increased the ventricle fills more during diastole and therefore stroke volume increases. If Psa increases then the ventricle empties against a higher pressure and stroke volume decreases. Of course cardiac output which is stroke volume x heart rate also depends on the heart period T.

2. Another simpler model of the ventricle

In this model the ventricle is models as a simple variable compliance. Below is the model for the left ventricle.



Here Pvp (pulmonary venous pressure) is the preload of the left ventricle, whereas in the previous model the preload was Pla. Pvp and Pla (left atrial pressure) are not very different anyway. Also Rav is the same as Rlv of the previous model. Rav is the viscous resistance of the left ventricle plus the resistance of the aortic valve, but resistance of the aortic valve is negligible compared to the viscous resistance of the ventricle.

The terms in the model are as follows:

- mv, av are mitral, and aortic valves
- Rmv is the resistance of the mv
- Rav is combined resistance of aortic valve and left ventricular viscous resistance
- Pas is systemic arterial pressure, or equivalently aortic pressure
- Pvp is pulmonary venous pressure which is almost equal to left atrial pressure

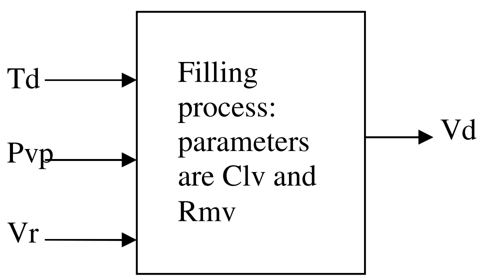
Plv is left ventricular pressure
Vlv is left ventricular volume

Typical values are
Rmv = 0.004 mmHg.sec/ml
Rav = 0.06 mmHg.sec/ml
Clv = 1/Slv where Slv is left ventricular stifffness and has the time course
Slv = SLD + SLS x sin(2Πt /(2Ts)) for 0 <= t <= Ts
= SLD for Ts <= t <= T
where SLD = 0.033
SLS = 1.5
where t is time and t = 0 is the start of systole
Ts is systole time and is 0.3 sec
T is heart period and is 0.8 sec
Td is diastole time and is 0.5 sec.

The ventricle fills with blood during diastole and empties by an amount, Vs, called the stroke volume, during systole. The filling and emptying dynamics of the ventricle can be studies separately.

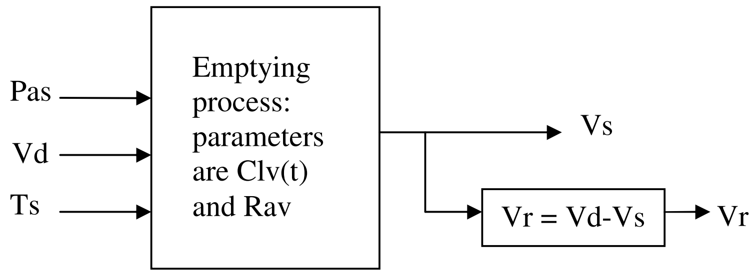
2.1. The Filling Process

During diastole the mitral valve is open, the aortic valve is closed, and the ventricle fills under the effect of Pvp. During diastole the ventricle is a compliance with Clv = 1/SLD.



Here Vd is the end-diastolic volume and is the volume arrived at at the end of diastole. Vr is the residual volume which remains in the ventricle at the end of systole, that is, it is the volume of the ventricle at the onset of diastole. Although Rmv is defines as the rsistance of the mitral valve, it also reflects the resistance of the atrium and the viscous forces of the ventricular mass.
Assuming Pvp is constant it is trivial to show that
$V_{lv}(t) = C_{lv}P_{vp} + (V_r - C_{lv}P_{vp})e^{-t/R_{mv}C_{lv}}$ and that $V_d = C_{lv}P_{vp} + (V_r - C_{lv}P_{vp})e^{-T_d/R_{mv}C_{lv}}$,
where $0 \le t \le T_d$ and $t = 0$ is the onset of diastole.

2.2.1. The Emptying Process



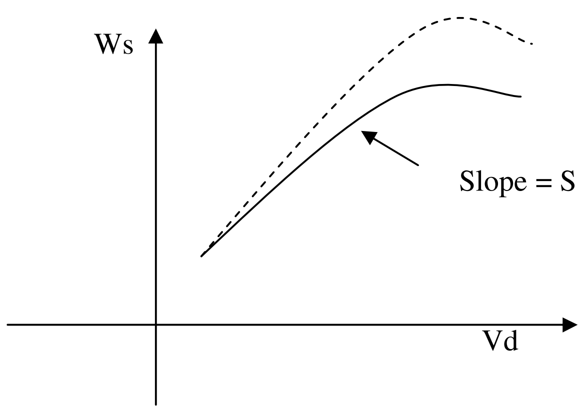
During systole the left vetricle empties against the after-load Pas, the opposing force. Also with the onset of systole the copliance Clv begins to decrease and thereby Plv increases. For a short while both valves are closed and this period is called the isovolumic phase during which the ventricle neither fills nor empties. When Plv exceeds Pas the aortic valve opens and the ventricle begins to empty. Another isovolumic phase occurs when Clv begins to increase and Plv falls below Pas.

During systole since Clv is time varying, the differential equations for Plv and Vlv are not simple to solve and a numerical procedure may be used.

Disregarding for the moment the exact time courses of Plv and Vlv we may just note that at the end of systole an amount of Vs is ejected, and a volume of Vr is left in the ventricle.

2.2.2. The Frank-Starling Law

Frank and Starling have found experimentally that the useful work done by the vetricle, Ws =Vs*Pas, has the following dependence



In the increasing portion Ws = S*Vd where S is a constant of proportionality and is called the "cardiac contractility index",i.e. a measure of the strength of the myocardium. It is known that the hormone "norepinephrine" increases s so that the dotted line in the figure is obtained. Thus the emptying function can be written as

Ws = Pas* Vs = S* Vd

and that

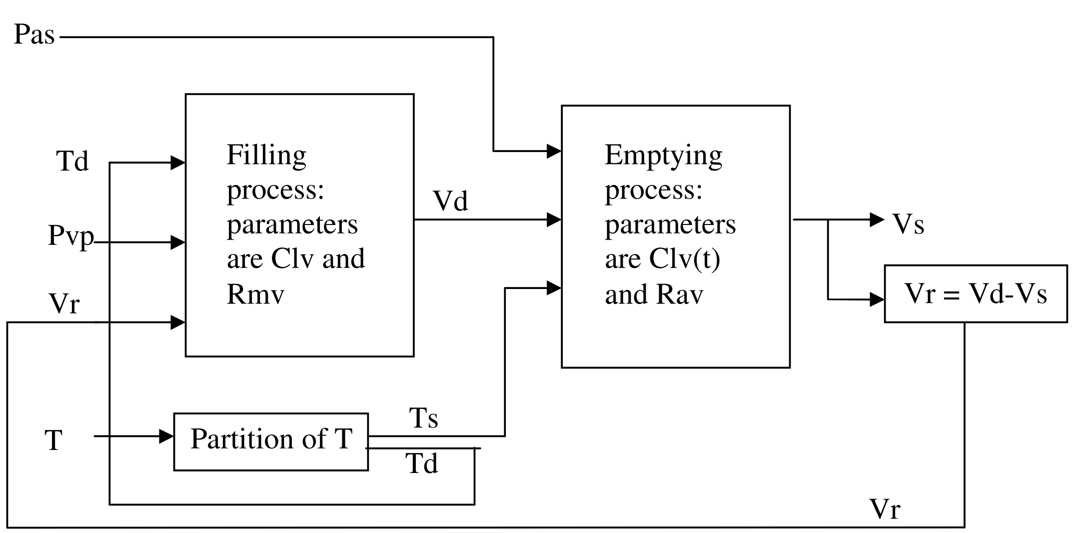
$$V_s = S\frac{V_d}{P_{as}} \qquad \textit{Frank-Starling Law}$$

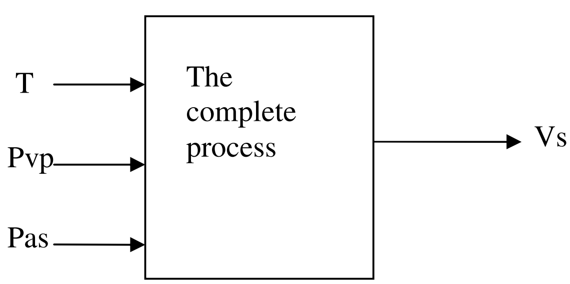
Thus the heart empties out a larger stroke volume is initially it has been filled more, and also if the after-load, pas, is less.

In the above figure the rising portion is called “the region of compensation” and this is where the heart usuyally works. The stauration-after region is called the “fatigue” region and this occurs if the the ventricle is over filled.

The Frank-Starling law is drawn from experimental results. However the two models that we have presented above can also be used to obtain the same law. If norepinephrine is applied then S increases and this correspons to an increase in SLS in the second model and an increase in ***Elv*** max in the Ursino model.

<u>2.3. The complete process – the overall behavior</u>





We have used the Ursino model with Rla = 25e-3 mmHg.s/ml
to understand the overall behavior of the ventricle**.** Results of the simulations are

a) For Pvp = 7 mmHg and T = 0.8 s

Psa	Vd	Vr	Vs	C.O.
65	105.24	38.9	66.34	4975.5
75	106.2	42.35	63.85	4788.75
85	107.15	45.8	61.35	4601.25

95	108.15	49.3	58.85	4413.75
105	109.1	52.9	56.2	4215
115	110.1	56.5	53.6	4020
125	111	60.1	50.9	3817.5

b) For Psa = 95 mmHg and T = 0.8s

Pvp	Vd	Vr	Vs	C.O.
3	66.7	49.3	17.4	1305
5	84.45	49.3	35.15	2636.25
7	108.1	49.3	58.8	4410
9	125.6	49.3	76.3	5722.5
11	140.9	49.4	91.15	6836.25
13	154.2	49.4	104.8	7860

c) For Pvp = 7 mmHg and Psa = 95 mmHg

HR	T	Vd	Vr	Vs	C.O.
65	0.92	114	49.3	64.7	4205
75	0.8	108.15	49.35	58.8	4410
85	0.706	101.75	49.4	52.35	4450
95	0.63	95.4	49.4	46	4370
105	0.57	89.7	49.4	40.3	4231
115	0.52	84.6	49.45	35.15	4042

As seen from the above tables as Psa is increased, while other parameters are kept constant, Vs and C.O. decrease as is the case with the Frank-Starling law. Basically against a larger after-load the ventricle cannot empty as much. As Vs decreases Vr increases and Vd also rises slightly as the ventricle fills more starting from a larger Vr.

If Pvp, the pre-load, increases, while other paramaters kept constant, Vs and C.O. increase. This happens mostly by an increase in Vd because the ventricle fills more with an increased pre-load. An increase in Vd causes an increase in Vs which is also in accordance with the Frank-Starling law.

If the pre-load and the after-load are kept constant while T is decreased we observe that Vd decreases and also Vs decreases. This is so because with decreased T, Td decreases more so than Ts and the filling time is less. A less filled ventricle empties less of course, again in accordance with the Frank-Starling law. Note that as Vs decreases C.O. increases first because the heart rate is increasing. However with very small heart periods Vd decreases so much that the increase in heart rate cannot compensate for the decrease in Vs, and the C.O. decreases.

If Rla = 2.5e-3 mmHg.s/ml is used the fall in CO is not observed in the physiological HR range. If Rla = 10e-3 mmHg.s/ml is used the fall in CO begins approximately at 125 beats per second which is a more realistic situation (see the solution of the 2nd problem in HW8).