Full-Wave and Approximate Solutions of Large Electromagnetic Scattering Problems

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***Abstract*—We report solutions of several large-scale electromagnetic scattering problems involving more than one billion unknowns. Full-wave solutions are provided using the parallel multilevel fast multipole algorithm (MLFMA). An out-of-core method is implemented for reducing the memory requirements of MLFMA solutions. Additionally, fast and approximate solutions using physical optics are provided.**

I. Introduction

The multilevel fast multipole algorithm (MLFMA) is used for efficiently obtaining full-wave solutions of electromagnetics problems involving arbitrary geometries. The algorithm has $\mathcal{O}(N \log N)$ time and memory complexity, where N is the number of unknowns [1]. As an alternative, physical optics (PO) can be used for approximate calculations of scattering from electrically-large objects with $\mathcal{O}(N)$ complexity.

In order to solve large-scale problems, we use MLFMA on distributed-memory architectures with the hierarchical partitioning strategy [2], which provides an excellent parallelization efficiency. Nevertheless, memory requirement for storing some data structures grows immensely when very large geometries are involved. As a remedy, an out-of-core method is implemented for utilizing the disk space to store the large data structures [3].

II. Out-of-Core and Parallel MLFMA

The out-of-core implementation writes the near-field interactions in the impedance matrix and the far-field patterns of the basis functions on disk, then it reads them in each matrix-vector multiplication in the iterative solution.

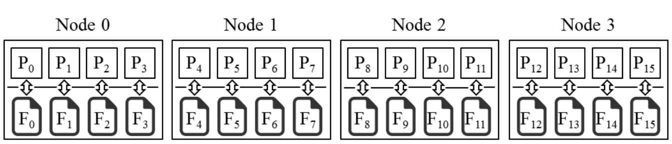

Fig. 1. The out-of-core MLFMA is employed on a four-node computer cluster with 16 processes. Assuming there is a single disk drive in each node, each disk drive handles the I/O jobs of four processes simultaneously. A process is shown with $\mathrm{P_i}$, where $\mathrm{i} \in \{0, 1, 2, ..., 15\}$ is the process ID, and a file is shown with $\mathrm{F_i}$. Each process owns a distinct file, i.e. $\mathrm{P_i}$ owns $\mathrm{F_i}$, and it names its file with a unique name in order to find the file whenever it is needed.

In the parallel implementation, each process has its own portion of the basis functions and the processes write their out-of-core data simultaneously. An implementation example involving a four-node cluster and 16 processes is depicted in Fig. 1.

III. Physical Optics

We expand the PO current on Rao-Wilton-Glisson functions and find the current coefficients by solving $\boldsymbol{x}^{PO}$ in the system of equations

$$\overline{\boldsymbol{Z}}^{PO}\boldsymbol{x}^{PO} = \boldsymbol{v}^{PO}. \tag{1}$$

A matrix element Z_{mn}^{PO} is the interaction between the n^{th} tesing and the m^{th} basis functions, i.e.,

$$Z_{mn}^{PO} = \int_{S_m} d\boldsymbol{r}\ \boldsymbol{t}_m(\boldsymbol{r}) \cdot \boldsymbol{b}_n(\boldsymbol{r}), \tag{2}$$

and a right-hand-side vector element v_m^{PO} is the m^{th} testing of the PO current, i.e.,

$$v_m^{PO} = \int_{S_m} d\boldsymbol{r}\ \boldsymbol{t}_m(\boldsymbol{r}) \cdot 2\hat{\boldsymbol{n}} \times \boldsymbol{H}^{inc}(\boldsymbol{r}), \tag{3}$$

where $\boldsymbol{t}_m$ and $\boldsymbol{b}_n$ are the m^{th} testing function and the n^{th} basis function, respectively, S_m is the domain of $\boldsymbol{t}_m$, and $\boldsymbol{H}^{inc}$ is the incident magnetic field with its associated normal vector $\hat{\boldsymbol{n}}$ of the surface S_m. We set $v_m^{PO}{=}0$ if S_m is not illuminated directly by the source.

Each testing function overlaps with at most five basis functions and therefore $\overline{\boldsymbol{Z}}^{PO}$ is a sparse matrix with $\mathcal{O}(N)$ non-zero elements and we solve the system iteratively.

IV. Numerical Results

Several scattering problems involving extremely-large conducting geometries are solved with 64 processes in a 16-node computer cluster with 2 TB memory. Each node is equipped with a solid-state disk drive for storing the out-of-core data. The geometries involve a sphere, a NASA Almond, and a stealth airborne target Flamme [4] geometries with the radius of 500λ and the length of 2104λ and 2402λ, respectively, where λ is the wavelength of the illuminating plane wave in free space. The geometries are discretized with the mesh size 0.09λ and the Bi-CGSTAB solver is employed along with a block-diagonal preconditioner for satisfying 1% residual error in the iterative solutions. The Almond and Flamme geometries are illuminated at 30° from their sharp ends on the azimuth plane with horizontal polarization, where 30° and 210° are their backscattering and forward-scattering angles, respectively.

Table I shows the CPU times and the memory requirements of the MLFMA solutions. The required disk space for employing the proposed out-of-core method is denoted in the table.

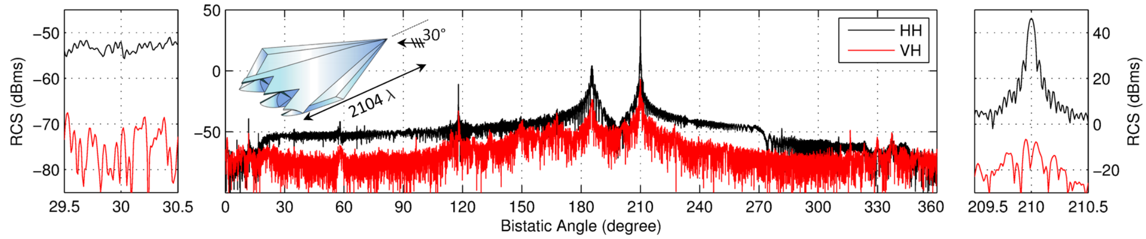

Fig. 2. RCS of the Flamme. The solution involves 1,338,909,696 unknowns. Co-polar and cross-polar RCS values are denoted with HH and VH labels.

TABLE I
SOLUTION TIMES AND MEMORY REQUIREMENTS

	Sphere	**Almond**	**Flamme**
CPU Time (hours)	57.0	46.7	57.7
Memory (terabyte)	1.2	1.9	2.0
Disk (terabyte)	2.1	2.2	2.7

The iterative solver performs 37, 39, and 38 iterations for the sphere, Almond and Flamme solutions, respectively. Taking less than one hour, the PO solutions are very fast compared to the MLFMA solutions, but the radar-cross-section (RCS) results are approximate as seen in Figs. 3 and 4.

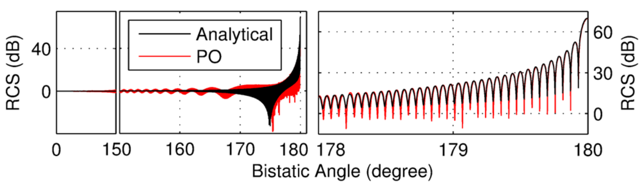

Fig. 3. RCS of the sphere in dB. The solution involves 1,109,280,768 unknowns. The computational RCS errors are 0.65%, 1.21%, and 2.63% in the 0°–30°, 0°–90° and 0°–180° bistatic angle sectors, respectively, where analytical Mie series solution is taken as reference.

Figure 3 shows the RCS of the sphere in dB and Fig. 4 shows the RCS of the NASA Almond in dBms. We observe that the PO results are in good agreement with the full-wave results near the backscattering and the specular-reflection angles, but they are approximate in general. The calculation method of the RCS error in Fig. 3 is provided in [5], where interested readers can compare their results with the full-wave solutions of the sphere and the NASA Almond problems.

Figure 2 shows the full-wave RCS results of the Flamme. The solution involves approximately 1.3 billion unknowns and it is the largest solution that we can solve within 2 TB memory. Without the out-of-core implementation, the largest Flamme that we could solve within 3 TB of memory has a length of 1640λ and the problem involves approximately 540 million unknowns. This evidence shows that the out-of-core implementation is successful in decreasing the required memory and providing extremely-large MLFMA solutions.

V. CONCLUSIONS

MLFMA solutions of large-scale problems, involving more than 1.1 billion unknowns, are reported. A parallel out-of-core implementation is used for reducing the MLFMA memory. We observe that PO results are in agreement with the corresponding full-wave results around the forward-scattering and specular-reflection angles. Therefore, it is a powerful tool for RCS calculations of electrically-large geometries when corresponding full-wave solutions are not available.

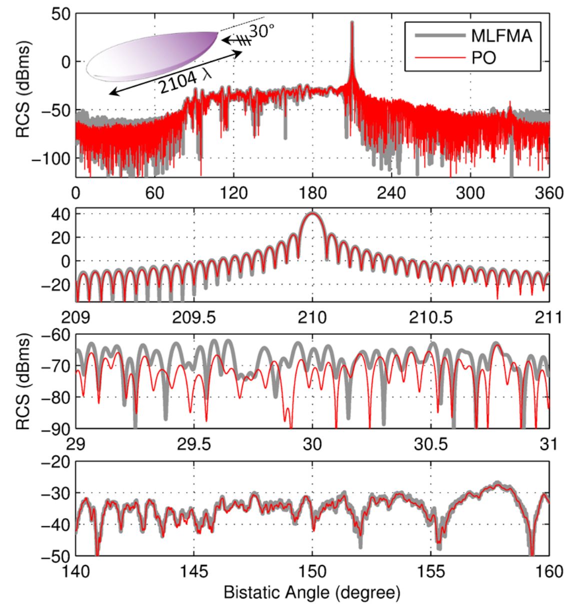

Fig. 4. Bistatic RCS of the NASA Almond. The MLFMA solution is plotted with the grey line as the reference, whereas the PO solution is plotted with the red line. We observe that the PO results are in good agreement with the MLFMA results around the forward-scattering (210°) angle and in the plateau near the specular-reflection (150°) angle, whereas it is approximate in other angles including the backscattering (30°) angle. The solutions involve 1,126,503,936 unknowns. The PO solution takes 51 minutes and it performs 13 CGS iterations in order to solve the system of equation displayed in (1) with 10^{-6} residual error.

ACKNOWLEDGMENTS

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