STRUCTURE OF MULTIPOLAR INTERNATIONAL SYSTEMS

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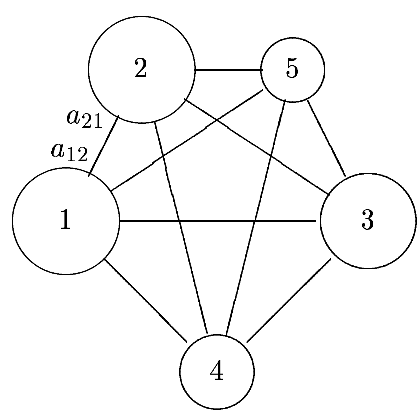

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Abstract

This paper proposes a formal model which clarifies the circumstances under which the conclusions of structural theory follow from its premises. The model incorporates a basic feature of a multipolar system: each unit apportioning its capability, allocating its resource, against the other units. The analysis focuses on various notions of equilibrium under allocation of resources. Unlike earlier formal studies of n-state systems, the model draws a careful conceptual distinction between state acts that are shaped by structural constraints, and state acts that are guided by specific motives.

We show that the depiction of an international system as composed of like units with specified capabilities is sufficient to determine the existence, or the lack thereof, of a wide range of equilibria among its units, with or without alliances. If the capability of one member is so large as to make it a hegemon, then equilibrium in many different senses can not exist. If, however, the system contains no hegemon, then a wide range of equilibria are possible.

Introducing a simple notion of security under allocation of resources, a strategic n-person game in which each state is independently maximizing its security is obtained. A Nash equilibrium of this security game turns out to be an allocation profile attaining as close an equality of bilateral allocations as possible for all states. Anarchy further manifests itself as a failure to coordinate at a particular allocation scheme when there are multiple Nash equilibria. As such, the model indicates that in an anarchic environment, attaining a bilateral equality of allocations is inherently dynamic and risky. Certain resource distributions are shown to trigger endogenous formation of alliances in accordance with the balancing imperative of Waltz. These results and a detailed analysis of alliances, whether exogenous or endogenous, indicate that the distinction between "a world in which equilibrium prevails" and "a world in which equilibrium is possible" should carefully be drawn in order to bring some clarity to (neo)realist notions of balance-of-power and balancing-act.

1 INTRODUCTION

In his influential book (1979) Kenneth Waltz outlines a structure theory for international politics. His main thesis is that structural forces working at systemic level are far more relevant to understand international politics than forces produced by influential statesmen, public pressure, or skillful foreign diplomacy. He identifies the ordering principle (anarchy), the function of units (like units called states), and the distribution of capabilities across the units (relative capabilities of states) as the structure of an international system. He further assumes that "the states seek to ensure their survival"(1979, p. 91) or put differently " the dominant goal of states is security"(1997, p. 915). This assumption together with the structural constraints leads to a balance-of-power theory the main proposition of which is known as the balancing imperative, "balances of power tend to form whether some or all states consciously aim to establish and maintain a balance, or whether some or all states aim for universal domination" (1979, p. 119). The means available to states in order to achieve a balance-of-power "fall into two categories: internal efforts (moves to increase economic capability, to increase military strength, to develop clever strategies) and external efforts (moves to strengthen and enlarge one's own alliance and shrink an opposing one)" (1979, p. 114).

While the controversies surrounding structural views of international politics continue, only meager effort is being made on developing mathematical models to base such discussions on. One must recall the main thesis of the structural theory: there are binding structural forces operating at the system level which "shape and shove" the behavior of the units of the system. Thus, to discover whether forces at the structural level are responsible for the behavior of states, one must first start without imposing any *a priori* motive or purpose on the states. Alternatively, one may try to identify those structural constraints that continue to operate under the widest set of possible motives.

Existing studies within the realm of structural realism attribute some *a priori* motives to states. Frequently, a game theoretic framework is adopted to view states as interacting units with prescribed preferences (possibly conflicting too) over outcomes. If we correctly understand what Waltz means by structure, we should then admit that game theoretic studies impose more on state actions than the structural theory deems necessary. Consequently, it is not an easy task to disentangle those conclusions that are due to structural forces from those that owe their origin to particular motives attributed to players.[1]

[1]Many of the formal (model-based) studies with the exception of Wagner (1986) and Niou, Ordeshook, and Rose (1989) are constrained to two-actor or effectively bipolar systems. While these have the merit of focusing the attention on simpler cases, they do not give clues as to the validity of conclusions reached in the extended cases of three or more actors. Waltz himself has constrained his analysis to bipolar systems because he found such systems to be more stable and because a similar analysis of systems having more than two (effective) states he anticipated to be too complex. Even in bipolar models, different assumptions about state preferences, reflected in chosen utility functions, lead to heated discussions of whether they conform to neorealist theory or not, see e.g. Grieco, Powell, and Snidal (1993). The need that the assumptions on motives should be made transparent in game theoretic studies of international systems has been forcefully brought forth by works of Powell, see Powell (1991, 1993) and Grieco, Powell, and Snidal (1993).

Waltz derives his balancing imperative by stipulating that among a state's list of goals, survival is the foremost. Different motives, such as straining for ever more power (classical realist motive), responding to a threat, expectation of easy gains lead to quite different behavior, such as bandwagoning, buck-passing, chain-ganging, and balancing of interests.[2] Each such alternative motive has found support within the realist school and has been argued, at some length, to be either more basic than or complementary to the motive of survival. Waltz's appeal to the rationale "to pursue whatever other goals [states] may have, they must first survive" (1997) seems to have but little effect. The unanswered question is not whether the survival motive is a realistic assumption, but rather, if so assumed, whether the balancing imperative is a necessary consequence of the system structure as claimed by Waltz.

The model we study assumes no more than the definition of structure in Waltz (1979). The system is comprised of n states with the distribution of resources, $r_1, ..., r_n$, or capabilities.[3] So far the model assumes like units called "states" with specified (relative) capabilities called "resources" and concurs with the last two components of structure as defined by Waltz. The novelty we introduce is allocating or targeting these resources to others, in contrast to directing total resources to one or a group of states at once.[4] How one should interpret the numbers r_i's[5], whether they can be taken to be infinitely divisible, and the implications of allowing no self allocation have been discussed at length in Niou, Ordeshook, and Rose (1989). In a truly multipolar environment, states do need to apportion their resources against the rest, albeit in the form of divided attention.[6] Moreover, since allocations can be anything between zero and the total resource endowment, r_i, the possibility of directing the whole resource to just one state or to a selective number of states at one time is not ruled out.

In Section 2, we first introduce an algebraic notion of equilibrium, called b-equilibrium, as equal bilateral allocations among all states. Figure 1 illustrates a five-state-system at b-equilibrium. The lines emanating from a state denote its respective allocations and the

[2]See Walt (1987), Christensen and Snyder (1990), and Schweller (1994).

[3]Resources are numbers but they would neither represent head-count of soldiers, nuclear warheads, nor annual GNP. Their relative values have a significance, but again there may not be much of a difference between resource values 1 and 0.8. Resources are quantities enabling us to attain qualitative conclusions only!

[4]Arthur Lee Burns (1958) seems to be the first to examine a balance of power under targeting of resources among three or more states which are "roughly of a size". The primary problem he examines is how alliances among some states make them more powerful against the third parties. Since he pays more attention to alliances and coalitions, he does not pursue the problem of attaining an equilibrium under targeting of resources and quickly turns to the study of alliances.

[5]To give a recent example, the conclusions reached by the guns-versus-butter model of Powell (1993) concerning the role of anarchy differ depending on whether the level of military technology is incorporated into the definition of resources or not. Waltz (1993) elaborates on how nuclear weapons limit force at the strategic level to a deterrent role and how they make alliances obsolete. These indicate difficulties associated with representing "capability" by a single number. Such difficulties are not peculiar to the theory of international systems. Representing the "mass" of a body or the "capital" of a firm by a number is not less problematic.

[6]Deutsch and Singer (1964) base their notion of stability in a multipower system on diminishing share of attention as the number of actors increases.

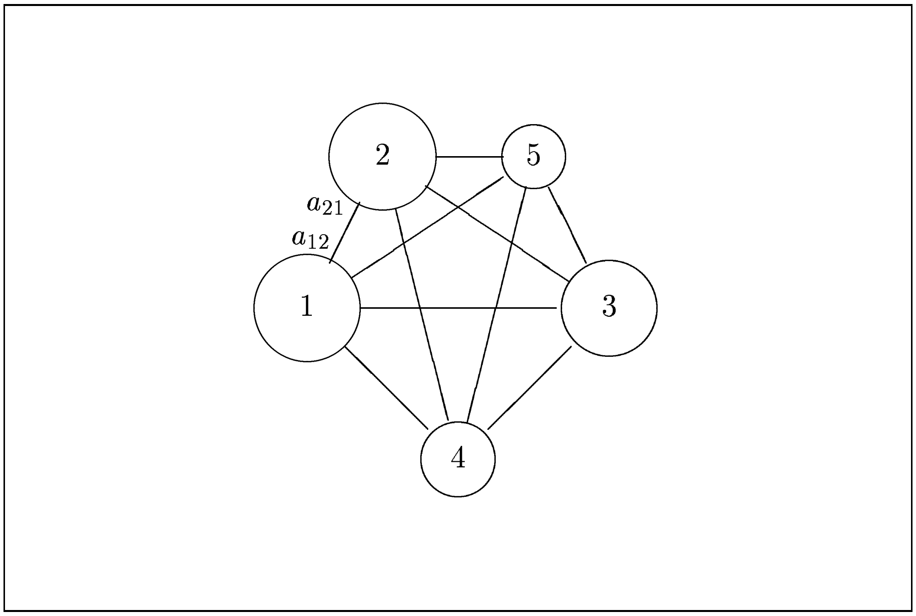

Figure 1: Bilateral-Equilibrium

sum of their values equals the total resource of that state. As two emanating lines are connected, this is interpreted as the equality of bilateral allocations. The conditions on resource distribution across states under which a b-equilibrium exists is next investigated. An answer is provided by Proposition 1 which also characterizes possible allocation profiles that constitute a b-equilibrium. A b-equilibrium is possible just in case there is no hegemon. It is unique if there are two or three states or if one state is a near hegemon. In case the number of states is four or larger and they contain no (near) hegemon, then there are infinitely many b-equilibria.

In Section 2.2, a simple definition of (individual) security is introduced and a strategic "security game" where each state independently maximizes its own security, or equivalently, minimizes its insecurity, is analyzed. Theorem 1 shows that if no hegemon exists, then every Nash equilibrium of the security game is a b-equilibrium, and *vice versa*. If there is a hegemon, then the Nash equilibrium is an allocation profile that is an approximate b-equilibrium. Thus, there are infinitely many Nash equilibria in the case of four or more states with no (near) hegemon. In such a case, anarchy as the lack of coordination implies that the attainment of a Nash equilibrium is a dynamic process, takes time, and is uncertain. In Section 2.3, we indicate a possible means for the refinement of Nash equilibria. We introduce the notion of a perfect equilibrium observing that a consistent "threat perception" by the states shrinks the set of available b-equilibria. A perfect equilibrium is a b-equilibrium where one of any two given states are consistently and equally emphasized through allocations of the rest. Proposition 2 shows that a perfect equilibrium exists whenever the resource distribution across the states is roughly uniform, when the weaker states are not so weak, and it is attained by a unique allocation profile. Thus, additional preferences to seeking security can and does single out a unique strategy among the infinitely many Nash strategies.

In Section 3, we investigate structural constraints on alliances. We mean by an alliance a broad range of subsets of states. A coalition is a subset of states acting in unison as an effective state. Section 3.1 draws upon the alliances that emerge due to structural constraints while the states play the security game of Section 2.2. Endogenous (spontaneous) alliances of differing solidarity are shown to emerge whenever one state approaches hegemony or when two or more states (for reasons external to the model) coalesce in a near hegemonic-coalition. The notion of alliance-equilibrium, abbreviated as a-equilibrium, introduced in Section 3.2 adopts the notion of bilateral-equilibrium of Section 2.1. It is defined as a scheme of internal and external allocations attaining a b-equilibrium inside every alliance as well as among alliances. Proposition 4 shows that structure allows an a-equilibrium, under any alliance configuration, just in case there is no global hegemon. While the nature of allocations attaining a bilateral and an alliance equilibrium are quite different, the existence conditions and the conditions for uniqueness remain the same. The main structural constraint against a-equilibrium is again the presence of a global hegemon. In alliances considered by Proposition 4, no *a priori* limitations are imposed on the level of internal or external allocations; they are, in a sense, extremely fragile. The strongest possible contract among alliance members is a reduction of their mutual allocations to zero which transforms them into a coalition. Section 3.3 examines the structural constraints in the presence of coalitions and defines a coalition-equilibrium, or c-equilibrium, as a bilateral-equilibrium among coalitions, where each coalition is treated as an effective state. We first note that a partition of the states into three or more coalitions which are in c-equilibrium is possible just in case a global hegemon does not exist. Next, focusing on weaker states, we show in Theorem 2 that the absence of a global hegemon is also a necessary and sufficient condition for a c-equilibrium to exist among some four or more coalitions one of which being the weakest state as a singleton. In case of three coalitions one of them being the weakest state, a c-equilibrium still exists provided the weakest state is not "too weak". The condition of "not being too weak" is made precise and indicates that such a notion of c-equilibrium coincides with the notion of system stability formalized by Niou and Ordeshook (1986) through their cooperative n-person game. Our structural analysis of a world in which coalitions can form indicate that any state seeking security will always act towards "breaking" any hegemonic-coalition that may be formed against it using all available means.

These formal results are recapitulated in Section 4 in an attempt to bring some clarity to realist and neorealist notions of balance-of-power and balancing-act. Based on the distiction between "a world in which equilibrium prevails" and "a world in which equilibrium is possible", there are two distinct meanings attributable to "a balancing-act". It may be an act towards "attaining an equilibrium when no hegemon exists" and it consists of a proper choice of an allocation profile by each state. It may be an act towards "making equilibrium possible when a hegemon exists" and mainly involves internal efforts Waltz mentions. As the existence of a hegemon does not permit any alliance-equilibrium either, forming alliances can not be qualified as an act towards making equilibrium possible. Instead, *dissolving* alliances can be qualified as a balancing act to end the hegemony of a coalition when one exists. A formal balancing imperative given in Section 4 encompasses a balance-of-power and a balancing-act in both of the above senses.

The following sections are organized in accordance with our two main aims of (i) examining the limitations imposed by pure structure and (ii) verifying the balancing imperative of Waltz. Sections 2.1, 3.2, and 3.3 give three structural notions of equilibrium, namely, b, a, and c-equilibria. These are introduced without any reference to any state preferences or motives. The main motivation for focusing on these notions and for designating them by "equilibrium" is that they are anticipated to be game-theoretic equilibria for a large class of games. The b-equilibria of Section 2.1 are indeed shown to be precisely the Nash equilibria of the security game introduced in Section 2.2.[7] The notions of consistent and perfect equilibrium of Section 2.3 are examined as refinements of Nash equilibria under additional motives to seeking security. Nonetheless, these two notions also illustrate how structure interacts with motives to constrain a state's behavior. Similarly, possible motives that yield a-equilibrium and c-equilibrium are also examined in Sections 3.2 and 3.3. The results of Sections 2.2, 2.1, and 3.1 confirm the balancing imperative and fulfill our second aim.

2 A FORMAL STRUCTURAL THEORY

Any model of as complex a system as the international politics appears unrealistic in its description of the system and thus incomplete in its conclusions. It is not uncommon that even the unapologetic theorist is carried away by a sense of urgency pressed by the current world events. Many however will admit that the field is yet too young and much patience and caution should be exercised before shedding light on complex real world problems. Gilpin (1981, p.3) remarks: "...until the statics of a field of inquiry are sufficiently well developed and one has a good grasp of repetitive processes and recurrent phenomena, it is difficult if not impossible to proceed to the study of dynamics." The complexity of the subject further complicated by the strong presence of human factor is repeatedly offered as an argument against abstractions. One wonders however whether a passenger plane is less complex, and whether one can even start examining a passenger plane without the abstract theory of gravity and the simple laws of aerodynamics. The model chosen below is as simple and as abstract as possible because this is a natural course the human mind follows when confronted with a complex task.

Consider then the following model of an international system consisting of n states $\mathcal{N} := \{1, ..., n\}$ with the distribution of endowed resources $r_1, ..., r_n$. State-i is assumed in full control of its own resource, r_i, but none of others. Each r_i is infinitely divisible and totally used up by apportioning among remaining states so that allocations add up to r_i. More concretely, our model of an $n-$state-system is the set of all $n \times n$ resource allocation matrices, with ij-th entry, a_{ij}, *allocation by i against j*, for $i \neq j$. As there is no self-allocation, matrices have zero diagonal entries, and the i-th row sums to the resource endowment, r_i, of state i. An *allocation profile of state-i* is the set of values

[7] A game-theory oriented reader may hence read Section 2.2 first, followed by Section 2.1.

$a_i := \{a_{ij} : a_{ij} \geq 0, \sum_{j \in \mathcal{N}-\{i\}} a_{ij} = r_i, \; j \in \mathcal{N} - \{i\}\}$. An *allocation profile* is a set $\{a_i : i \in \mathcal{N}\}$ which is a set of allocation profiles of all states. It is easier to visualize an allocation profile as a resource allocation matrix. Its i-th row corresponds to an allocation profile of state-i. Given four states with resources $r_1 = 1, r_2 = 0.9, r_3 = 0.8, r_4 = 0.7$, the matrices below give three different allocation profiles:

$$A_1 = \begin{bmatrix} 0 & 0.5 & 0.2 & 0.3 \\ 0.3 & 0 & 0.2 & 0.4 \\ 0.3 & 0.1 & 0 & 0.4 \\ 0.1 & 0.1 & 0.5 & 0 \end{bmatrix}, A_2 = \begin{bmatrix} 0 & 0.2 & 0.1 & 0.7 \\ 0.2 & 0 & 0.7 & 0 \\ 0.1 & 0.7 & 0 & 0 \\ 0.7 & 0 & 0 & 0 \end{bmatrix}, A_3 = \frac{1}{6}\begin{bmatrix} 0 & 2.3 & 2.0 & 1.7 \\ 2.3 & 0 & 1.7 & 1.4 \\ 2.0 & 1.7 & 0 & 1.1 \\ 1.7 & 1.4 & 1.1 & 0 \end{bmatrix}. \quad (1)$$

2.1 Bilateral-Equilibrium

We shall focus on a particular scheme of allocation: A *bilateral-equilibrium*, or simply, *b-equilibrium* is an allocation profile which is constrained by the equality of mutual allocations, i.e., by $a_{ij} = a_{ji}, \; \forall \; \{i, j\} \subset \mathcal{N}$. A b-equilibrium is represented by a symmetric resource allocation matrix. The matrix A_1 in (1) represents an allocation profile which is not a b-equilibrium, whereas A_2, A_3 give two different b-equilibria.[8] We now consider the following questions concerning a b-equilibrium:

Q1. For a given resource distribution, $r_1, ..., r_n$, is there a b-equilibrium in the n-state-system?[9]

Q2. What are the possible b-equilibria when one exists?

If $n = 2$, the answers are trivial: a b-equilibrium exists if and only if the two states have equal amounts of resources which are fully directed against each other.[10] If $n \geq 3$, then Q1 and Q2 are answered by Özgüler, Güner, and Alemdar (1998). Before we summarize this result below, some definitions are in order: If a state's resource endowment strictly exceeds half of the total resource in the system, or equivalently, if it is strictly greater

[8] Our focus on "bilateral-equilibrium" stems from our concern with "balance" which recalls among others "equality". One can also single out other allocations of interest such as a number of states being in a particular "dis-equilibrium" and ask, as we will ask for equilibrium, whether structure allows such an allocation. Focusing on equilibrium and its relation to structure does not mean that the states are viewed *a priori* as units concerned with equilibrium.

[9] This is equivalent to asking whether there is a symmetric matrix in the set of all resource allocation matrices.

[10] The fact that the whole model collapses into a rather trivial situation in the case of two states is a valid criticism of the level of abstraction present in the model. Whether such an abstraction is useful or not depends on how much it can tell us about a world of three or more states. Quite a bit as we hope to illustrate below! Once the present model is thoroughly studied, some of its abstractions can then be discarded. As an example, a slight extension of the present model, obtained by allowing internal allocations and by constraining the allocations into prespecified intervals, is a similar effort to Niou and Ordeshook (1989), and tells us nontrivial facts even about a world of two states, see Özgüler, Güner, and Alemdar (1999).

than the total resource of the rest, we call it a *global hegemon*. A *near global hegemon* is defined as a state which owns exactly half of the total resources in the system.

Proposition 1. *In an n-state-system with a given resource distribution, $r_1, ..., r_n$, the following hold:*

(i) If there is a global hegemon in the system, then no b-equilibrium exists.

(ii) If there is no global hegemon, then a b-equilibrium always exists.

(iii) In the absence of a global hegemon, there is a unique b-equilibrium for $n = 2$ and $n = 3$. For $n > 3$, b-equilibrium is unique if there is a near global hegemon, and infinitely many if there is none.

Thus, a b-equilibrium can exist within a wide range of system structure: from one where resources are equally distributed to the other extreme where one state controls exactly half of the resources in the system. The substantive parts of Proposition 1 are statements (ii) and (iii) characterizing the b-equilibria. In Özgüler, Güner, and Alemdar (1998), explicit expressions for one possible b-equilibrium is given for any nonhegemonic n-state-system. Here, we examine the possible b-equilibria for $n = 2, 3, 4$.

Recall that a_{ij} denotes the allocation of state i to state j. In a bipolar system,[11] no hegemon exists if and only if $r_1 = r_2$ in which case the only possible b-equilibrium is $a_{12} = a_{21} = r_1$. In a tripolar system, no hegemon exists if and only if $r_i \leq r_j + r_k$ for every permutation (i, j, k) of $(1, 2, 3)$. Under this condition, the only possible b-equilibrium is obtained by $a_{jk} = a_{kj} = (r_j + r_k - r_i)/2$ for every permutation (i, j, k) of $(1, 2, 3)$.

The smallest size system which, in general, admits an infinity of different b-equilibria is the system of four states. In order to describe such profiles, let us number the states such that

$$r_1 \geq r_2 \geq r_3 \geq r_4 > 0. \tag{2}$$

Thus, the system is nonhegemonic if and only if $r_1 \leq r_2 + r_3 + r_4$. Under this condition, the set of all possible b-equilibria are as follows:

$$\begin{aligned}
a_{12} = a_{21} &= \tfrac{1}{2}(r_1 + r_2 - r_3 - r_4) + y, \\
a_{13} = a_{31} &= \tfrac{1}{2}(r_1 + r_3 - r_2 - r_4) + x, \\
a_{23} = a_{32} &= \tfrac{1}{2}(r_2 + r_3 + r_4 - r_1) - (x + y), \\
a_{14} = a_{41} &= r_4 - (x + y), \\
a_{24} = a_{42} &= x, \\
a_{34} = a_{43} &= y,
\end{aligned} \tag{3}$$

[11] Two distinct meanings are attached to the term "bipolar world". The first is that, although the world contains many states, the capabilities of all states can be assumed to be null relative to the capabilities of two strong states. An alternative meaning is that the world is partitioned into two subsets of states (coalitions) with the members in each subset acting in unison with capabilities combined. Here, the term is used in the former sense. The latter is considered in the section on alliances.

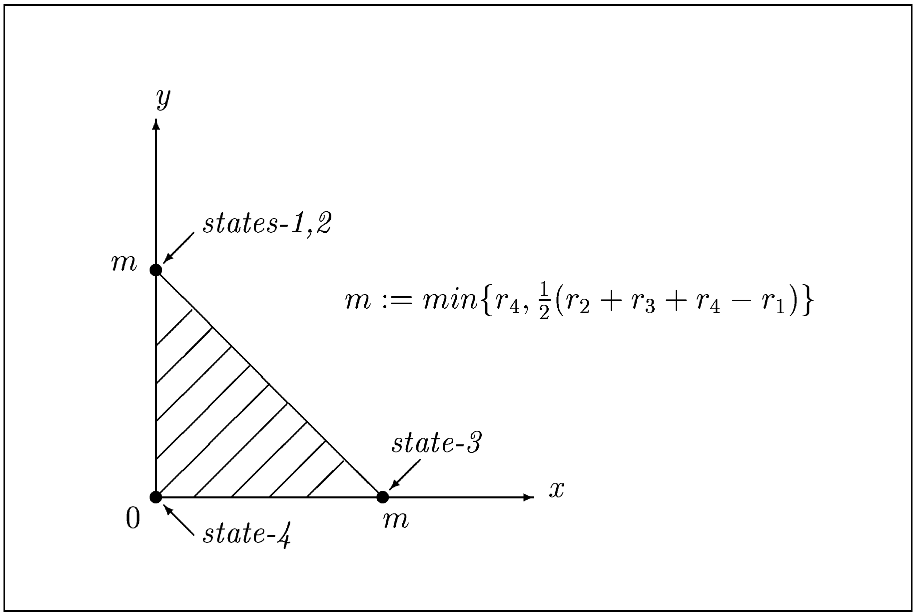

Figure 2: Set of all parameters and possible misinformed choices

for any x, y satisfying

$$0 \leq x + y \leq min\{r_4, \frac{1}{2}(r_2 + r_3 + r_4 - r_1)\}. \tag{4}$$

The triangle described by (4) is shown in Figure 2. Every point inside the shaded triangle yields a different b-equilibrium via (3). In (1), the b-equilibrium represented by A_2 is obtained by the point $(x, y) = (0, 0)$ on the triangle and A_3 by $(x, y) = (\frac{1.4}{6}, \frac{1.1}{6})$. By way of verifying Proposition 1, note that if the first state is a near hegemon, then $r_1 = r_2 + r_3 + r_4$ and the constraint (4) indicates that the triangle degenerates to a point $(x, y) = (0, 0)$. Using this fact in (3), the only possible b-equilibrium is obtained as: $a_{12} = a_{21} = r_2, a_{13} = a_{31} = r_3, a_{14} = a_{41} = r_4, a_{ij} = 0 \; \forall \; i \neq 1, j \neq 1$. Notice that all weak states allocate their total resources against the near global hegemon and none against each other.

The results above can be readily generalized to n-state-systems. As the resource gap between the strong and the weak widens, allocations of the weaker states against the stronger have to necessarily increase for the attainment of a b-equilibrium. The allocations of weaker states among themselves necessarily tend to zero as one among the stronger approaches to being a near hegemon. If, for instance, state-1 is a near hegemon, then the unique b-equilibrium dictates that states $2, ..., n$ direct their total resources against state-1 and none against each other.[12]

[12]We further elaborate on such structural constraints below in Section 3.1.

2.2 A Security Game

Following Waltz, let us now assume that a basic motive of every state is to ensure its security.[13] In the model under consideration, this would mean that a state chooses its allocations with an eye towards maximizing its security. We define the *bilateral security* of a state-i against state-j by

$$s_{ij} = a_{ij} - a_{ji}.$$

With this definition $s_{ji} = -s_{ij}$ so that one state's security is another's insecurity. The *individual security* of state-i is defined as its minimum bilateral security $\min_{j \in \mathcal{N}-\{i\}}\{a_{ij} - a_{ji}\}$. If state-$i$ chooses its allocations, $a_{ij},\ j \in \mathcal{N}-\{i\}$, so as to maximize its security, then the problem facing state-i can be posed as follows: Choose $a_{ij} \geq 0$ where $\sum\limits_{j \in \mathcal{N}-\{i\}} a_{ij} = r_i$ such that the individual security of state-i

$$u_i = \min_{j \in \mathcal{N}-\{i\}}\{a_{ij} - a_{ji}\} \tag{5}$$

is maximum for any given $a_{ji};\ j = 1, ..., i-1, i+1, ..., n$. If every state is a security maximizer in the above sense, then we end up with a noncooperative infinite game with the strategy space of each state i defined over its allocation profile a_i. Before stating the main result which describes a Nash equilibrium, let us note that $u_i = -\max_{j \in \mathcal{N}-\{i\}}\{a_{ji} - a_{ij}\}$, so that maximizing the security is the same as minimizing the maximum individual "insecurity". The adopted utility definition is thus symmetric with respect to security and insecurity.

Theorem 1. *Consider the n-person strategic game with utilities (5).*

(i) If there is no hegemon, then an allocation profile is a Nash equilibrium if and only if it is a b-equilibrium. Otherwise, it is a Nash equilibrium if and only if the hegemon directs its resource against the rest in such a way as to leave its bilateral security uniformly the same. The remaining states take full action against the hegemon and none against each other. [14]

[13] Waltz (1979) does not seem to make any distinction between the motive of survival and the motive of security. If a state perceives a breach to its security as a threat to its survival, then the distinction indeed disappears. Zakaria (1998) criticizes Waltz for being ambiguous on the exact nature of state motive. The point of departure in such criticisms is Waltz's phrase "at a minimum, seek their own preservation and, at a maximum, drive for universal domination" (1979, p. 118) which, it is argued, imposes not one but a continuum of motives on states, Legro and Moravcsik (1999). In our interpretation, Waltz sees the motive of security as a *common denominator* of any motive they may care to have. He makes this clear by remarking on several occasions that in order to pursue any other goals, a state must first survive.

[14] If state-1 is a hegemon, then a Nash equilibrium is an allocation profile in which $a_{1j} = r_j + \frac{1}{n-1}(r_1 - \sum_{t=2}^{n} r_t),\ a_{j1} = r_j, a_{ij} = 0,\ i, j = 2, ..., n$. This is an *approximate b-equilibrium* in a well-defined sense (See Özgüler, Güner, and Alemdar (1998)). In the bordering case of the state-1 being a near hegemon, Nash equilibrium coincides with the b-equilibrium described at the end of Section 2.1.

(ii) An allocation profile is a Nash equilibrium if and only if it is a solution of the optimization problem[15]

$$\max_{a_{ij}} \min\{u_1, ..., u_n\} \tag{6}$$

where a_{ij}'s are nonnegative and $\sum_{j \in \mathcal{N}-\{i\}} a_{ij} = r_i$ *for all* $i \in \mathcal{N}$.

(iii) Every Nash equilibrium is strongly Pareto efficient, i.e., there is no allocation profile which achieves at least as large securities as Nash securities of all states with better security for at least one state.

A direct proof of Theorem 1 based on the best response functions of the states is given in the Appendix.

Observe that a Nash equilibrium always exists. Whenever there is a hegemon or a near hegemon, or $n \leq 3$, there is a unique Nash equilibrium. Otherwise, there are infinitely many Nash equilibria. Moreover, in the absence of a hegemon, every Nash equilibrium in the security game is a b-equilibrium of Proposition 1 and *vice versa.* [16] When there is a hegemon, security maximizing Nash allocations leave remaining states totally secure among themselves and equally insecure vis-a-vis the hegemon. Thus, the notion of b-equilibrium developed in Section 2.1 can be interpreted within a game theoretic context: In their quest for maximum individual security, states are led collectively to as close a b-equilibrium as possible.

The nonuniqueness of Nash equilibrium has disturbing consequences. Consider the system of four states and assume that no hegemon or a near hegemon exists. By Theorem 1, independent security maximization by each state implies a general behavior towards attaining a b-equilibrium whereas nothing specific is predicted concerning the actual process of reaching a b-equilibrium.[17] Every choice of parameter values (x, y) in the triangle of Figure 2 gives a Nash equilibrium. Suppose that the four states, each targeting to reach an equilibrium, choose the indicated points in Figure 2, i.e., states 1 and 2 choose $(x, y) = (0, m)$, state-3 chooses $(x, y) = (m, 0)$, and state-4 chooses $(x, y) = (0, 0)$. Under such choices, the formulae in (3) give $a_{13} < a_{31}$ and $a_{14} < a_{41}$ although equality is attained in every other case. Consequently, a Nash equilibrium is not reached in spite of the fact that each state chose its allocations towards achieving an equilibrium. Each state needs to know not only the resource values, but also the amounts targeted against itself by every other state to reach an equilibrium.

The nonuniqueness of Nash equilibrium necessitates a coordination process to choose a unique point in the triangle of Figure 2. Anarchy, understood as the lack of a central

[15] This is an L_∞-optimization problem, see the proof of (ii) in the Appendix and e.g. Chvátal (1983).

[16] In the sequel, the word "equilibrium" when used without any further qualification will mean both Nash equilibrium and b-equilibrium, provided the world contains no hegemon.

[17] A usual interpretation for the notion of Nash equilibrium is that it captures a "steady state" of a game played repeatedly without any strategic links between plays, Osborne and Rubinstein (1994).

(government) authority in the international system, hinders this. [18] In case of multiple equilibria, the anarchic nature of the international system implies that *achieving an equilibrium is a necessarily dynamic, iterative, process and involves much uncertainty* as states lack prior knowledge as to which equilibrium, among many, the other states are choosing.[19]

Burns (1958), Waltz (1979, p. 135), and others have underscored the importance of "seeking certainty" along with the basic motive of seeking security. The uncertainty which is caused by complexity of a multipolar system is usually indicated by referring to the number of decisions required by the maker of any decision in a system of n states, which is $n(n-1)/2$ and happens to be half the number of allocations. Our model points to a more fundamental uncertainty that arises in the absence of a central authority when there are multiple equilibria.[20] It is entirely conceivable that each individual state acts towards an equilibrium, yet the collective outcome is one of disequilibrium. In the situation depicted in Figure 2, state-4 does not know whether the other states have chosen their allocations towards an equilibrium or a disequilibrium. The anarchy and the nonuniqueness of equilibrium, hence imply an uncertainty of a different type than the ones usually indicated among security seeking states. Anarchy gives rise to a "false perception of intentions" if there are multiple equilibria.

Closing, we ask: Would a slightly different utility definition than (5) yield a quite different Nash equilibrium? Our conjecture is that so long as each utility penalizes bilateral insecurities of a state, b-equilibrium allocations will continue to be Nash equilibria of the respective game in a nonhegemonic system. In a hegemonic system, the Nash payoffs of all states except the hegemon's will continue to be zero and hegemon's payoff will still have a positive value. This conjecture is supported by the results in Özgüler, Güner, and Alemdar (1998) concerning the optimization problem of Theorem 1.ii. [21]

[18] The central role played by the notion of anarchy and its implications have been extensively discussed in the literature. Powell (1994), among others, analyzes two different formulations of anarchy; first, the lack of a central government, second, the availability of the use of force to the states. He finds that without an explicit articulation of the strategic environment, the assumption of anarchy by itself does not yield the behavioral implications (such as the balancing act) usually attributed to it. The lack of a central authority is a common feature of both meanings of anarchy.

[19] By contrast, albeit in a somewhat contrived fashion, if we were to imagine a supreme authority coordinating states to target their resources towards a specific equilibrium, then irrespective of whether the equilibrium is unique or nonunique, the attainment of equilibrium is instantaneous and may only be delayed by other mechanisms external to the structural model. Examples of such mechanisms are an opposition to the authority, internal moves by states towards increasing their resource levels, and formation of alliances or coalitions.

[20] Burns (1958) realizes however that uncertainty has also some relation to resource distribution in the system when he points out that the uncertainty is eliminated in case a "world dominion" exists, p. 502.

[21] When the L_∞-norm (6) is replaced by an L_2 or L_1 norm, this changes only the best utility of a hegemon.

2.3 From Consistent to Perfect Equilibrium

Faced with a multiplicity of Nash equilibria, do states have any incentive to meet at one particular profile? Is there a distinct Nash equilibrium among many? An affirmative answer requires the identification of further motives of states. A reduction in the possible equilibria can not be obtained unless players' preferences are modified.

We now show that some equilibria in the simple security game can be eliminated, possibly all the way to one, if the member states possess consistent views as to which of any given two states poses a greater security threat. To illustrate, let us reconsider the case of four states with the specified resource distribution in (2). Suppose now that a_{ij}, $\{i,j\} \subset \{1,2,3,4\}$, in addition to being a b-equilibrium, further satisfy

$$sign\{a_{31} - a_{32}\} = sign\{a_{41} - a_{42}\}. \tag{7}$$

Thus, $a_{31} > a_{32}$ if and only if $a_{41} > a_{42}$ so that whenever state-3 emphasizes state-1 so does state-4 and *vice versa*.

While one natural cause of such an emphasis may be that state-1 has more resource than state-2, which in and of itself may be a source of threat, there may as well be other reasons such as state-1 being more hostile than state-2, state-2 being geographically more distant than state-1, or state-1 having a better offensive capability than state-2.[22]

Whatever the cause of it, let us suppose the sign equality (7) holds. We show that this will drastically cut down the set of possible Nash equilibria. In fact, using (3) in (7) indicates that parameters x and y must further satisfy $r_2 + r_4 - r_1 \le 2x + y \le r_4$. If the resource distribution is such that $r_2 + r_3 \ge r_1 + r_4$, then $m = r_4$, $c = r_2 + r_4 - r_1$ and the admissible allocations consistent with (7) are obtained by parameters (x, y) inside the trapezoid T with corners $(0, m), (0, c), (c/2, 0), (m/2, 0)$ as shown in Figure 3. If $r_2 + r_3 < r_1 + r_4$, then a similar region is obtained. This reduction in potential profiles is possible thanks to the consistent views of states 3 and 4 about 1 and 2.

If all states are unanimous in their views as to which of any given two states is more threatening, i.e., if for every pair of states $\{k, l\}$,

$$sign\{a_{ik} - a_{il}\} = sign\{a_{jk} - a_{jl}\}, \tag{8}$$

for every $i, j \in \mathcal{N} - \{k, l\}$, then all possible *consistent equilibria* are those obtained by parameters (x, y) inside the small trapezoid $ABCD$ in Figure 3.

Depending upon a given resource distribution, this trapezoid might degenerate into a point or to a triangle. For instance, in our 4-state-system a unique equilibrium obtains if and only if $r_1 = r_2 = r_3$ or $r_2 = r_3 = r_4$. If, $r_1 - r_2 \ge m, r_2 - r_3 \ge m/2, r_3 - r_4 \ge m$, then the

[22] How such differences cause insecurity among other states has been subjected to intensive examinations. On offense/defense debate see Jervis (1978) and Van Evera (1998) and on a formal treatment of the role of geography see Niou and Ordeshook (1989).

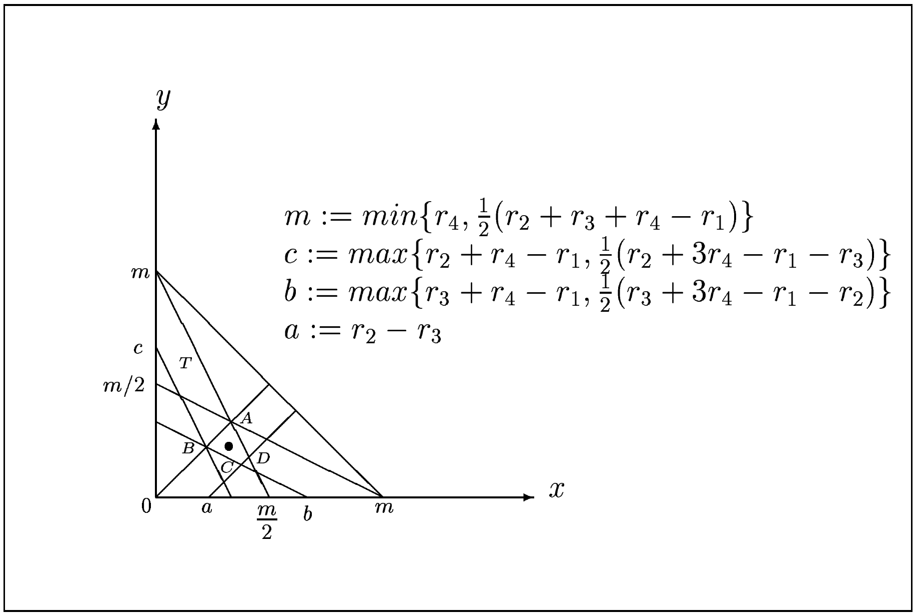

Figure 3: Region of Consistent Equilibria

triangle with corners $A, (0,0), (m/2, 0)$ results. Thus, a unique consistent equilibrium is possible if at least three states have equal resources. As the resource imbalance gets larger, the set of possible consistent equilibria enlarges. Recall, however, that as the resource imbalance gets larger, state-1 is also approaching to being a near hegemon. Consequently, the set of possible profiles itself is shrinking, i.e., the triangle in Figure 2 is diminishing in size. In effect, for any given resource distribution, the set of consistent equilibria is a "small" subset of b-equilibria.

Encouraged by such a reduction in multiple equilibria, let us now imagine a stronger consistency constraint on equilibria. In an n-state-system with $n \geq 3$, let the equilibrium allocations further satisfy the following: For any given pair of states $\{k, l\}$,

$$a_{ik} - a_{il} = a_{jk} - a_{jl}, \tag{9}$$

for every $i, j \in \mathcal{N} - \{k, l\}$, i.e., the given emphases are not only consistent, but they are equal as well. We call an allocation profile to be a *perfect equilibrium* if it is a b-equilibrium and the allocations of every pair of disjoint subsets $\{i, j\}$ and $\{k, l\}$ of $\mathcal{N}$ satisfy (9). The following result is proved in Özgüler, Güner, and Alemdar (1998) and gives the condition for a perfect equilibrium to exist together with the nature of allocations constituting a perfect equilibrium.

Proposition 2. *In a world of n states with $n \geq 3$,*

(i) a perfect equilibrium exists if and only if the sum of resources of the two weakest states

is not less than the average resource of the remaining states.[23]

(ii) If this condition holds, then there is a unique perfect equilibrium. The perfect equilibrium consistently emphasizes the stronger among any given pair of states.[24]

A perfect equilibrium is in particular a consistent equilibrium since (9) implies (8). Thus, in the case of four states for instance, the point (x, y) which yields the perfect equilibrium is located inside the small trapezoid as indicated by the dot in Figure 3. Among the allocation matrices in (1), A_3 corresponds to perfect equilibrium obtained by parameters $(x, y) = (\frac{1.4}{6}, \frac{1.1}{6})$.

In case $n = 3$, perfect equilibrium is the same as the unique b-equilibrium. The perfect equilibrium concept also conforms to common sense. Imagine, for instance, an n-state world, $n > 3$, where resources are uniformly distributed and recall that there are infinitely many Nash equilibria. A particularly intuitive Nash equilibrium for states with the same resource endowments is that they apportion their resources equally against the others. This indeed (by fn. (24)) is the unique perfect equilibrium of Proposition 2 for equal distribution of resources. In case $n = 4$, every choice of parameters in the triangle in Figure 2 gives a Nash equilibrium and the centroid (intersection of the medians) of the triangle gives the unique perfect equilibrium. As for the case of unequal distribution of resources, the common sense tells us that allocations against a stronger state should be relatively more than the allocations against a weak one. This again is borne out by Proposition 2.ii.

A further desirable property of perfect equilibrium is that it is singled out by a *comparative dynamic consideration*. Imagine now a small change in resources, $r_i, i \in \mathcal{N}$, that results in a small change in the system structure. If an equilibrium still exists sufficiently close to *status-quo ante*, when so perturbed, then there is no *a priori* reason for states to drastically change their equilibrium. That is, under such circumstances it is reasonable to assume that states will smoothly vary their allocations. Suppose then that we focus on those Nash equilibria that are continuous and differentiable functions of resources.[25] To be more specific, we consider allocations, a_{ij}, $\{i, j\} \subset \mathcal{N}$, that satisfy

$$\frac{\partial a_{ij}}{\partial r_l} = \frac{\partial a_{ik}}{\partial r_l}, \; \forall \, \{j, k\} \subset \mathcal{N} - \{i, l\}, \; \forall \, i, l \in \mathcal{N}. \tag{10}$$

This condition requires that state-i change its allocations to j and k by the same amount whenever there is a change in either its own resource, r_i, or a change in the resource of some other state l, r_l. The following result shows that a b-equilibrium that fulfills this regularity condition is the perfect equilibrium.[26]

[23]If $\{n-1, n\}$ are the two weakest states, then $r_n + r_{n-1} \geq \frac{1}{n-2} \sum_{t=1}^{n-2} r_t$. This condition implies the lack of a global hegemon so that it need not be separately stated.

[24]The perfect equilibrium is given by $a_{ij} = \frac{1}{n-1} (r_i + r_j - \frac{1}{n-2} \sum_{t \in \mathcal{N} - \{i,j\}} r_t) \;\; \forall \;\; \{i, j\} \subset \mathcal{N}$. Thus, it satisfies $a_{ij} - a_{ik} = (r_j - r_k)/(n-2)$ for any $\{j, k\} \subset \mathcal{N} - \{i\}$.

[25]Every a_{ij} is given by $a_{ij} = f_{ij}(r_1, ..., r_n)$, where f_{ij} is continuous with respect to each r_k and each partial derivative $\frac{\partial f_{ij}}{\partial r_k}$ exists and is continuous.

[26]Any dynamic model of a world of n-states must make some assumptions as to how allocations are

Proposition 3. *In a world of n states, if allocations are continuous and differentiable functions of resources, then the following holds: a b-equilibrium satisfies (10) if and only if it is a perfect equilibrium.*

A proof of Proposition 3 is given in the Appendix.

To sum up, recall that there are, in general, multiple Nash equilibria in four or more state systems. A *consistent* equilibrium exists in the absence of a hegemon and it can be considered as a first refinement of Nash equilibrium. A specific consistent equilibrium, *perfect* equilibrium, exists if the resource distribution is fairly uniform in the system, namely, if *the weaker are not too weak.* Perfect equilibrium is unique whenever it exists so that its attainment does not involve the transient stages that one finds in attaining a Nash equilibrium.

3 ALLIANCES

The term *alliance* is used to refer to any collection of states which may be bound by a formal agreement, by an informal understanding, or by a common view of the outsiders in an n-state system. Alliances are associations among a subset of states with alignments at the one extreme end, and coalitions at the other. A particularly weak alliance association is an *alignment* where members act similarly towards the outsiders with no commitment to reduce their mutual allocations inside the alliance. An example to an alignment is provided by the consistent equilibrium discussed at the beginning of Section 2.3. There, the views of states 3 and 4 coincide on which of the remaining states, 1 and 2, to emphasize in an equilibrium; $\{3, 4\}$ is an alignment against the states 1 and 2. The strongest form of alliance association, on the other hand, is a *coalition* whereby members are bound not to allocate any resource against each other.[27] A coalition agreement by fixing internal allocations to zero also eliminates uncertainty in the restricted sense of Section 2.2 among its members.

The self-help assumption of the structural theory goes beyond its usually emphasized aspect of anarchy as the lack of a central authority.[28] It also implies a reluctance to form alliances even when pressed by necessity. According to Waltz (1979, p. 107) "The inter-

updated in the face of changes in resources. The particular assumption (10), together with the assumption of smoothness, implies a perfect equilibrium. An alteration in (10), such as the introduction of a different proportionality constant than unity in the equalities, would single out another allocation profile. An argument that may be offered in favor of (10) is that it is a "non-discriminating" rule of updating allocations, since an increment in a resource r_i equally influences every allocation of state-i and equally influences allocations a_{jt} for all $j \neq i, t \neq i$.

[27] The use of the term "alignment" here is in line with the definition of Russett and Starr (1992, p. 91). The terms "alliance" and "coalition" have been used interchangably by many. Usually, the binding agreements are of a dynamic nature which can not be fully considered by the static model here. For example, Snyder's definitions of alliance and alignment focus on "expectations" among members, Snyder (1990). Allies still allocating some resources against each other indicate that they have both conflictive and cooperative interests. See Schroeder (1976, pp. 227-62).

[28] There are attempts to show that anarchy does not necessarily imply self-help, see e.g., Wendt (1992).

national imperative is "take care of yourself!"" and "Even with the greatest of external pressure, the unity of alliances is far from complete." (p. 167) He repeatedly points out to "managerial difficulties" any alliance will have to face. Grieco (1990) specifies two barriers to cooperation: enforcement problem and distribution of relative gains. Nonetheless, alliance formation is frequently observed in multipolar environments. The origin of alliances has always been a focal point in theory of international relations and many formal and empirical studies seek it deviating from or enhancing neorealist assumptions.[29] Regardless of what compels states to form alliances, one can examine structural limitations at work in their presence.

We first focus, in Section 3.1, on alliances that are endogenous to our model under the motive of security. Next, in Sections 3.2 and 3.3, we consider alliances that may be formed for reasons external to the model and which involve agreements on the nature of allocations among its members. The agreements may or may not impose a reduction in internal allocations. Alliance members may agree to allocate against each other possibly a fraction of what they would have, had they not been in the same alliance. Alternatively, they may act to reduce uncertainty by specifying a particular equilibrium inside the alliance. We will consider each alternative.

3.1 Endogenous Alliance Formation

A security maximizing state may find itself in an alliance with others for purely structural reasons. One such instance is when the strongest state in the system is a near hegemon or a hegemon. In Section 2, we have shown that security maximization leads to a coalition among $n-1$ states against the strongest. When the resource gap between the strong and the weak states widens, similar constraints arise for the weaker with differing strengths. We now illustrate such endogenous formation of alliances in a world of four states without a hegemon under two different circumstances: (i) the strongest state is close to being a near hegemon and (ii) two states form a strong alliance (for reasons external to our model). Suppose (2) holds and the states maximize their security. Then, by Theorem 1, Nash equilibria are described by (3) and (4).

(i) Suppose $r_1 = r_2 + r_3 + r_4 - 2\epsilon$ for some nonnegative number ϵ. As ϵ approaches zero, state-1 approaches to the position of a near global hegemony. The allocation profile (3) becomes $a_{12} = a_{21} = r_2 + y - \epsilon$, $a_{13} = a_{31} = r_3 + x - \epsilon$, $a_{23} = a_{32} = \epsilon - (x + y)$, $a_{14} = a_{41} = r_4 - (x + y)$, $a_{24} = a_{42} = x$, and $a_{34} = a_{43} = y$. The constraint (4) takes the form $0 \leq x+y \leq \min\{r_4, \epsilon\}$. As $\epsilon \to 0$, it is clear that $a_{ij} \to 0$ for all $i, j = 2, 3, 4$. Hence, states $2, 3, 4$ find themselves in an increasingly stronger alliance as state-1 approaches to being a near hegemon.

(ii) Consider the alliance $\{1, 2\}$. Since $r_1 \geq r_3$ and $r_2 \geq r_4$, the total resource of this alliance is greater or equal to half the total world resource. Suppose the states 1 and 2 reduce their mutual allocations towards its minimum possible value. By (3), this is

[29] Bueno de Mesquita and Lalman (1992) and Walt (1987) are among the recent influential studies.

equivalent to $y \to 0$. This necessitates in (3) that for a b-equilibrium $a_{34} = a_{43} \to 0$; forcing a coalition between the states 3 and 4.

In a four-state-world endogenous formations of strong alliances essentially consists of the cases (i) and (ii) above. For larger n, additional cases need be considered. The principle however remains the same: *If one state is close to being a hegemon, or if a group of states are enjoined in a strong alliance, then system structure necessitates that tight alliances or coalitions be formed among the rest.*

3.2 Alignments

Formally, we now consider a system of n states partitioned into $l \leq n$ alliances $\mathcal{A}_i$, $i = 1, ..., l$ which are nonempty and disjoint subsets of $\mathcal{N} = \{1, ..., n\}$ such that $\mathcal{A}_1 \cup ... \cup \mathcal{A}_l = \mathcal{N}$. Internally, each alliance $\mathcal{A}_i$ is a system of n_i states. Externally, each alliance is perceived as a unit or as an effective-state by other alliances. A fraction I_i of the total alliance resource

$$r(\mathcal{A}_i) := \sum_{j \in \mathcal{A}_i} r_j$$

of $\mathcal{A}_i$ is internally and the remaining fraction E_i is externally targeted so that $E_i + I_i = r(\mathcal{A}_i)$. The *internal endowment* I_i and the *external endowment* E_i are composed of fractions of the resource of each member state. Thus, for each $i = 1, ..., l$, we have

$$I_i = \sum_{j \in \mathcal{A}_i} \alpha_{ij} r_j, \; E_i = \sum_{j \in \mathcal{A}_i} (1 - \alpha_{ij}) r_j \tag{11}$$

for some $\alpha_{ij} \in [0, 1] \;\; \forall \, j \in \mathcal{A}_i$.

Given a fixed alliance configuration of l alliances, an *alliance-equilibrium*, or simply an *a-equilibrium*, is a set of l internal and one external allocation profiles such that *(i)* each $\mathcal{A}_i$ as a system of n_i states is in equilibrium for $i = 1, ..., l$ and *(ii)* the system of l alliances, viewed as effective-states with resources E_i, $i = 1, ..., l$, is in equilibrium. Figure 4 illustrates two alliances in equilibrium in a five-state-system. Note that the system is in equilibrium when each alliance is in both internal and external equilibrium. For the case $l = 2$, external endowments E_1, E_2 are necessarily allocations of $\mathcal{A}_1$ and $\mathcal{A}_2$ against each other. When $l \geq 3$, allocations of any alliance $\mathcal{A}_i$ to other $l - 1$ alliances are nonnegative and sum up to E_i.

Consider the 4-state-system with resources $r_1 = 1, r_2 = 0.9, r_3 = 0.8, r_4 = 0.7$ of Section 2 and two alliances $\mathcal{A}_1 = \{1, 2\}$, $\mathcal{A}_2 = \{3, 4\}$. Two different a-equilibria, with respect to this alliance configuration, are represented by the internal and external resource allocation matrices $A^i_{1,int}, A^i_{2,int}, A^i_{ext}$ for $i = 1, 2$:

$$A^1_{1,int} = \begin{bmatrix} 0 & 0.4 \\ 0.4 & 0 \end{bmatrix}, A^1_{2,int} = \begin{bmatrix} 0 & 0.2 \\ 0.2 & 0 \end{bmatrix}, A^1_{ext} = \begin{bmatrix} 0 & 1.1 \\ 1.1 & 0 \end{bmatrix},$$

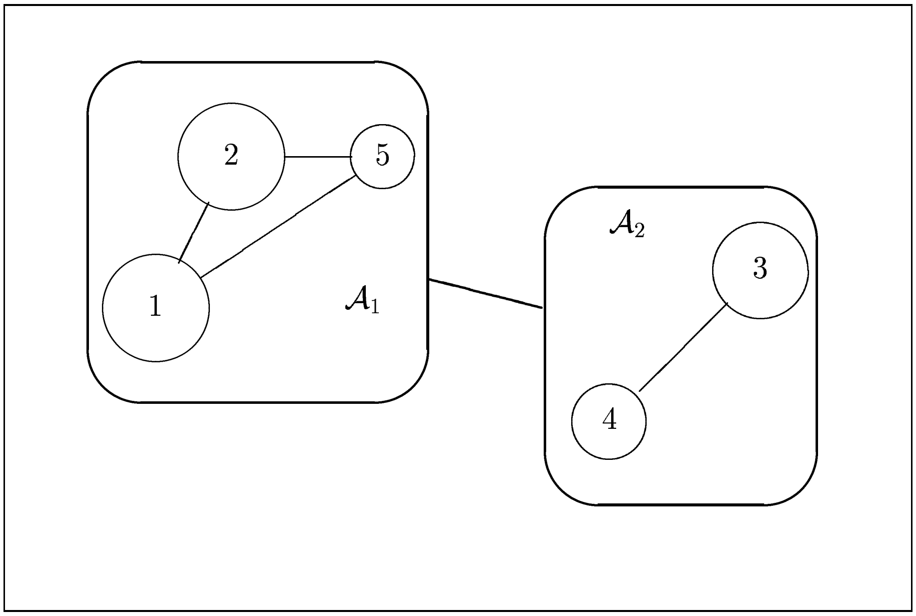

Figure 4: Alliance-Equilibrium

$$A^2_{1,int} = \begin{bmatrix} 0 & 0.2 \\ 0.2 & 0 \end{bmatrix}, A^2_{2,int} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}, A^2_{ext} = \begin{bmatrix} 0 & 1.5 \\ 1.5 & 0 \end{bmatrix}. \tag{12}$$

The first set of allocations are obtained with $E_1 = E_2 = 1.1, I_1 = 0.8, I_2 = 0.4$ and the second set of allocations with $E_1 = E_2 = 1.5, I_1 = 0.4, I_2 = 0$. Since each alliance contains two members only, the values of α_{ij}'s in this example are fixed for each choice of an internal endowment.

In the sequel, it will be helpful to define a *local* hegemon to be a state in an alliance with resource exceeding the sum of resources of the other members of that alliance. Further, let $\mathcal{A}_k$ be called a *hegemonic-alliance* if $r(\mathcal{A}_k)$ is strictly more than the sum of the total resources of the other alliances.

We now ask:

Q3. Given a system of n states and an alliance configuration, does there exist an a-equilibrium?

Q4. What are the possible a-equilibria when one exists?

It is not difficult to see that *every bilateral-equilibrium gives an alliance-equilibrium for any given alliance configuration.* As an example, the set of allocations in (12) are derived from the b-equilibrium represented by the matrix A_2 in (1).[30] Thus, the absence of a

[30] Given a b-equilibrium by a resource allocation matrix A, l internal and one external allocation matrices of an a-equilibrium can be obtained by the submatrices of a partition of A, see Özgüler, Güner, and Alemdar (1998).

global hegemon may be taken to imply that an a-equilibrium is possible for any alliance configuration. Furthermore, the fact that an a-equilibrium imposes no restriction on how the external endowment of an alliance $\mathcal{A}_i$ to another alliance $\mathcal{A}_j$ should be distributed among its members may give the impression that an a-equilibrium is less restrictive than a b-equilibrium. It turns out, however, that bilateral and alliance equilibria both exist under the same necessary and sufficient condition. The difference is in the allocation profiles. The following result is proved in Özgüler, Güner, and Alemdar (1998).

Proposition 4. *In an n-state-system with a given resource distribution, $r_1, ..., r_n$, the following hold:*

(i) If there is a global hegemon, then a-equilibrium does not exist for any alliance configuration.

(ii) If there is no global hegemon, then there always exists an a-equilibrium for any given alliance configuration; any b-equilibrium yields an a-equilibrium. The following particular external endowments also exist and allow an a-equilibrium: For $i = 1, ..., l$

$$E_i = \begin{cases} 0 & \text{if } \mathcal{A}_i \text{ has no local hegemon,} \\ r(\mathcal{A}_i) & \text{if } \mathcal{A}_i \text{ is not a hegemonic-alliance and has a local hegemon,} \\ \sum_{j \neq i} r(\mathcal{A}_j) & \text{if } \mathcal{A}_i \text{ is a hegemonic-alliance and has a local hegemon.} \end{cases} \tag{13}$$

(iii) Suppose there is no global hegemon. There is a unique a-equilibrium (all internal and external equilibria are unique) if and only if either $n \leq 3$ or a near global hegemon exists.

For any given alliance configuration, the resource distribution among the units avails an a-equilibrium just in case there is no *global* hegemon. Whether some alliances contain a local hegemon or not and whether they are hegemonic-alliances or not has no relevance to the existence of an a-equilibrium. Such possibilities have an influence on the a-equilibrium itself as examplified by (13) in the same way as the existence of stronger or weaker states influences the b-equilibrium as discussed in the previous section. The proposition also shows that the particular alliance configuration chosen is of no relevance to the existence of an a-equilibrium; whether some alliances are formed by all weak members or all strong members does not alter the possibility of attaining an a-equilibrium. This happens as we have not constrained either the internal or the external endowment of any alliance. The alliances have been treated as alignments.

Is there any advantage of forming an alignment over confronting the rest of the states alone? Does entering an alignment make a state more secure or powerful than before? The answer strongly depends on the motives attributed to states and on the structure. Suppose first that there is no global hegemon and each state seeks to maximize its security. Since an equilibrium with or without alliances is possible only if there is no global hegemon, at a first glance, there seems to be no advantage of engaging in a formation of alignment in the absence of a hegemon. There is, however, a main motivation for forming alignments even in the absence of a hegemon and it originates from the motive of reducing uncertainty. If some states have the additional motive of reducing uncertainty in a multipolar environment, they may decide on *an agreement to meet at a specified bilateral-equilibrium inside the alliance.* Such an agreement, being a step away from anarchy, makes the attainment of an equilibrium inside the alliance instantaneous. In a coalition this takes the form of an agreement to meet at the equilibrium of zero allocations and automatically determines the total external endowment of the coalition as the total resource of that coalition. If such agreements can be enforced, then a hegemonic-coalition once formed achieves positive security for every coalition member. In its weakest form, the agreement neither fixes the total external endowment nor how this total is formed by individual contributions. The agreement simply specifies a function assigning an internal profile to any given external endowment of that alignment. Given any external (and therefore, internal) endowment, the function determines a profile of internal allocations. A stronger agreement would be to fix the internal (and therefore, the external) endowment of the alliance as in the case of coalitions. Such an agreement would make the alliance an effective-unit with resource equal to the external endowment of that alliance. The existence of an a-equilibrium is then answered by Proposition 4, where such alliances are treated as a single state. [31]

3.3 Coalitions

Alliance association, even in its weakest form, demands some commitment from its members. Even when the association is motivated by the expediency to reduce uncertainty inside an alliance, a similarity in world view is needed. If it is further desired to fix internal allocations at zero, a coordination mechanism is needed to monitor and enforce members' compliance. In a self-help environment forming and maintaining a coalition is more of an exception than a rule unless it becomes a structural imperative as we discussed in Section 3.1. Though of a temporary nature, coalitions may emerge through various motives and structural limitations at work on such coalitions need to be examined.

In the model under consideration, if one or more of the alliances are coalitions, then the external endowment of coalitions are the sum of the resources of member states. This situation is depicted in Figure 5, where there are two coalitions with resources $r(\mathcal{C}_1) = r_1+r_2$ and $r(\mathcal{C}_2) = r_3+r_4+r_5$. The system of five states is thus *reduced* to a system of two effective states having resources $r(\mathcal{C}_1)$ and $r(\mathcal{C}_2)$. Whether or not a b-equilibrium among these effective states exists is determined by an application of Proposition 1. If the world consists of l fixed coalitions, then this is a world of l effective states and a

[31] Further benefits of forming alliances originate from considerations external to the model. Our model assumes that the whole resource of each country is used up by targeting against the other states. In the presence of alliances, an alternative is possible. If all states in an alliance i are friends, then the internal endowment in alliance $\mathcal{A}_i$ can be considered as the spare resource that can be consumed for domestic needs in that alliance. The advantage of being in an alliance is then clear. Depending on whether $\mathcal{A}_i$ is facing friendly or hostile alliances, the domestic consumption in $\mathcal{A}_i$, and hence in its member states, can be accordingly enjoyed. Weak states in $\mathcal{A}_i$ may particularly enjoy the benefits of being in an alliance provided the stronger states contribute more to the external endowment E_i. For a collective good theory of disproportionate cost sharing in alliances see Olson and Zeckhauser (1966).

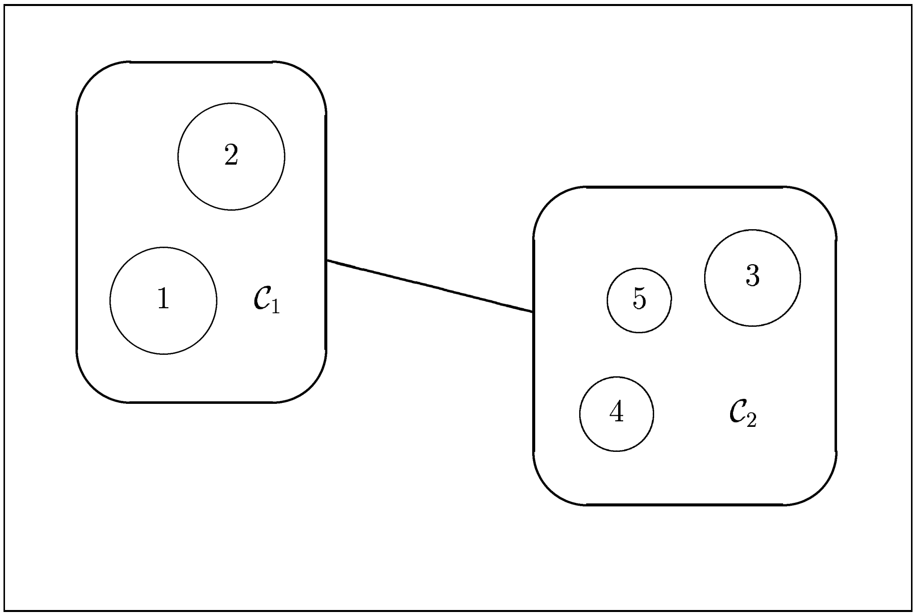

Figure 5: Coalition-Equilibrium

coalition-equilibrium, or a *c-equilibrium*, is defined as a bilateral-equilibrium where each coalition is treated as a state of resource $r(\mathcal{C}) = \sum_{i \in \mathcal{C}} r_i$. For the simple reduced system in Figure 5, a c-equilibrium exists if and only if $r(\mathcal{C}_1) = r(\mathcal{C}_2)$. The general case of a system of l alliances some of which are constrained to be coalitions is handled by Proposition 4 by representing each coalition $\mathcal{C}$ by a trivial alliance of one member (a singleton) having a resource equal to $r(\mathcal{C})$. Note that, if the world consists of l coalitions, then the definitions of security in Section 2.2 applies to these effective states. The (individual) security of each coalition member is set equal to the security of the coalition.

We first answer the following question for a world of n states with resources

$$r_1 \geq r_2 \geq ... \geq r_{n-1} \geq r_n > 0. \tag{14}$$

Q5. Given $2 \leq l \leq n$, does there exist a partition of the world into l coalitions such that a c-equilibrium exists?

Using Proposition 1 applied to l states, Q5 can be reformulated as: Does there exist a partition of the world into l coalitions $\mathcal{C}_1, \mathcal{C}_2, ..., \mathcal{C}_l$ none of which is a hegemonic-coalition? It immediately follows that the lack of a global hegemon is again a necessary condition for such an equilibrium to exist since, in view of (14), if state-1 is a global hegemon, then the coalition into which it enters (or itself) will be a hegemonic-coalition. It turns out that if $l \neq 2$, then the lack of a global hegemon is also sufficient for the existence of any c-equilibrium. *There exists a c-equilibrium for any l such that $3 \leq l \leq n$ if and only if there is no global hegemon among the n states.*[32] In the case $l = 2$, Q5 is the same as

[32]To see the sufficiency of the condition, suppose that state-1 is not a hegemon. Let $1 \leq k \leq n/2$ be

asking whether the world can be partitioned into two coalitions with equal resources, i.e., whether a coalition with resource equal to one half of the total world resource exists. An affirmative answer requires a very special relationship among the resources $r_1, ..., r_n$.[33]

Given that a coalition-equilibrium as well as a bilateral-equilibrium exist under the same condition, namely, the absence of a global hegemon, what can be the advantage of joining a coalition? One answer is that certain states may want to reduce uncertainty, the same motive given for forming alliances. Further insights as to the (im)possibility of coalitions can be obtained by focusing on the security concerns of weak states. Consider the following question from the point of view of state-n, the weakest in the system:

Q6. Given (14) and $2 \le l \le n$, does there exist a partition of $\{1, ..., n\}$ into l disjoint coalitions $\mathcal{C}_1, ..., \mathcal{C}_{l-1}, \mathcal{C}_l = \{n\}$ such that the world is in c-equilibrium?

If the answer to Q6 is affirmative, then the weakest state in the world is not too weak in the sense that it can face the world alone and still hold a possibility of c-equilibrium in at least one coalition configuration. If the answer is negative, then the weakest state must seek coalitions with stronger states for achieving a c-equilibrium, i.e., must ask the question Q5. Although the question Q6 is most relevant to state-n, it also concerns all the other states. If the answer to Q6 is affirmative and the weakest state is not so weak, then all the other states $n-1, n-2, ...$ are also not so weak and they hold the same possibility of attaining an equilibrium under a coalition configuration while standing alone.[34] This further motive for coalitional behavior thus comes from an evaluation of the system structure from the viewpoint of smaller states. Note that, *if the answer to Q6 is affirmative, then the actual coalitional behavior implied for state-n (and for any other state) is "breaking" any possible opposing hegemonic coalition into l smaller coalitions!*

We now proceed to answer Q6. Note, first, that if $l = 2$, then a c-equilibrium with state-n acting alone is possible if and only if $n = 2$ and $r_1 = r_2$, i.e., if and only if the world consists of two states with equal resources. We hence assume that $l \ge 3$ in the following.

Theorem 2. *Given positive resources satisfying (14), the following hold:*

(i) There exists a partititon of the world into $l \ge 4$ coalitions $\mathcal{C}_1, ..., \mathcal{C}_{l-1}$, $\mathcal{C}_l = \{n\}$ in c-equilibrium if and only if state-1 is not a global hegemon.

---

such that $r_1 + ... + r_k \le r_{k+1} + ... + r_n$ and $r_1 + ... + r_{k+1} > r_{k+2} + ... + r_n$. Such a k exists since state-1 is not a hegemon. Now let $\mathcal{C}_1 := \{k+1\}$, $\bar{\mathcal{C}}_2 := \{1, ..., k\}$, $\bar{\mathcal{C}}_3 := \{k+2, ..., n\}$. It follows that $\mathcal{C}_1, \bar{\mathcal{C}}_2, \bar{\mathcal{C}}_3$ is a partition of the world into 3 coalitions having no hegemonic-coalition. If $l > 3$, then partition $\bar{\mathcal{C}}_2$ and $\bar{\mathcal{C}}_3$ separately further into sub-coalitions to obtain $\mathcal{C}_2, ..., \mathcal{C}_l$ satisfying $\mathcal{C}_2 \cup ... \cup \mathcal{C}_l = \bar{\mathcal{C}}_2 \cup \bar{\mathcal{C}}_3$. In the new world of l coalitions, a hegemonic-coalition still does not exist and, by Proposition 1, a c-equilibrium is possible.

[33] A two-coalition-equilibrium exists if and only if "$n = 2 \Rightarrow r_1 = r_2$", "$n = 3 \Rightarrow r_1 = r_2 + r_3$", "$n = 4 \Rightarrow r_1 = r_2 + r_3 + r_4$ or $r_1 + r_4 = r_2 + r_3$", and so on for larger n.

[34] If the answer to Q6 is affirmative, then state-n is the coalition $\mathcal{C}_l$ and there is no hegemonic-coalition. Let us imagine state-n switching places with any other state k in some coalition $\mathcal{C}_j$. Since $r_k \ge r_n$, the coalition $\mathcal{C}_j$ does not have more resource than before and the resource of all the other coalitions remain unchanged. Moreover, state-k is not a hegemon itself since $\mathcal{C}_j$ was not a hegemonic-coalition when it included state-k. Therefore, there is still no hegemonic-coalition in the new coalition configuration and state-k can achieve an equilibrium standing alone as well.

(ii) There exists a partititon of the world into three coalitions $\mathcal{C}_1, \mathcal{C}_2$, $\mathcal{C}_3 = \{n\}$ in c-equilibrium if and only if the resource of state-n is not less than the difference between the resources of two coalitions of the closest resource values among all coalitions obtained by partitioning $\{1, ..., n-1\}$ into two coalitions.[35]

A proof is given in the Appendix.

Suppose there is no global hegemon. Assume that the weakest state stands alone. A c-equilibrium always exists for some three or more coalitions facing the weakest state. If only two coalitions exist against the weakest state, then the possibility of an equilibrium depends on the resource level of that state. If the weakest state is not so weak that the excess resource of the stronger of the two coalitions can not overwhelm the weakest state, then a c-equilibrium exists by Theorem 2.ii and it is unique by Proposition 1.iii. Otherwise, forming a third coalition becomes necessary for a c-equilibrium.

The conditions which render the system stability[36] of Niou and Ordeshook (1986) are exactly the same conditions required for the existence of a c-equilibrium with three coalitions one of which is the weakest state as a singleton. By Proposition 1, an equilibrium is possible among (some) two coalitions and the weakest state, i.e., there is a disjoint partition of $\mathcal{N}$ into three nonempty subsets $\mathcal{N} = \{n\} \cup \mathcal{C}_1 \cup \mathcal{C}_2$ such that the reduced world of three states with resources $r_n, r(\mathcal{C}_1)$, $r(\mathcal{C}_2)$ is at b-equilibrium, if and only if

$$\begin{aligned}
&r_n \leq r(\mathcal{C}_1) + r(\mathcal{C}_2), \\
&r(\mathcal{C}_1) \leq r_n + r(\mathcal{C}_2), \\
&r(\mathcal{C}_2) \leq r(\mathcal{C}_1) + r_n.
\end{aligned} \tag{15}$$

We now note that, since $r_n > 0$, one of the second and third inequalities in (15) must be strict. If the second inequality in (15) is strict, then state-n on entering the coalition $\mathcal{C}_2$, causes a stronger coalition than $\mathcal{C}_1$. If the third inequality is strict, then state-n on entering the coalition $\mathcal{C}_1$, causes a stronger coalition than $\mathcal{C}_2$. In other words, if an equilibrium with two coalitions and the state-n is possible, than state-n holds a card of making a difference in the relative strengths of the two coalitions. State-n is essential to making any of the two coalitions winning. Figure 6 shows a situation where state-5, the weakest state, is able to attain a c-equilibrium with some two coalitions.

Above considerations together with Remark 3.7 and Theorem 3.2 in Niou, Ordeshook, and Rose (1989) imply the equivalence of the following statements: (i) A c-equilibrium among state-n and some two coalitions exist. (ii) The system of n states is stable. (iii) For every $i \in \mathcal{N}$ and for every threat against i, i has a viable counterthreat. (iv) Every state is essential to at least one minimum winning coalition. Since c-equilibrium among

[35] Formally, $r_n \geq \min_{\mathcal{C}_1, \mathcal{C}_2} |r(\mathcal{C}_1) - r(\mathcal{C}_2)|$, $\mathcal{C}_1 \cup \mathcal{C}_2 = \{1, ..., n-1\}$, $\mathcal{C}_1 \cap \mathcal{C}_2 = \emptyset$. This condition implies, but is not in general implied by, the absence of a global hegemon. For $n = 4$, the condition is equivalent to state-1 not being a hegemon *and* $r_4 \geq r_2 + r_3 - r_1$; for $n = 5$, it is equivalent to no hegemon *and* $r_5 \geq \min\{|r_2 + r_3 + r_4 - r_1|, |r_2 + r_3 - r_1 - r_4|$. A general equivalent explicit expression is not clean enough to be given here.

[36] The definition of system stability is that no country is eliminated in the cooperative n-person game of Niou, Ordeshook, and Rose (1989).

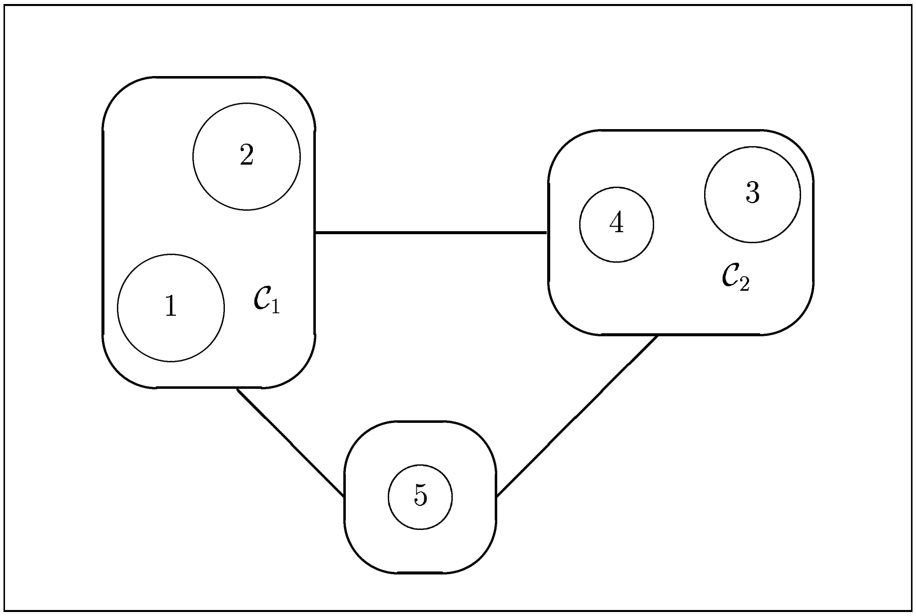

Figure 6: System stability as c-equilibrium

state-n and two coalitions is only one of many possible structural notions of equilibrium, the game posed in Niou and Ordeshook (1986) *and the particular preferences imposed on the states* yield the existence of a specific c-equilibrium as the characterization of system stability.

3.4 Hegemonic-Coalitions

If the world has a global hegemon, then other states can not possibly form a hegemonic-coalition and the strong position of the hegemon prevails also in a world with coalitions. While coalitions can not eliminate hegemony, they can create one. A hegemonic-coalition is a mathematical possibility in a world without a global hegemon. This possibility may make the idea of a coalition attractive to every state since each would enjoy positive security in a hegemonic coalition. In fact our analysis of coalitions have not yet focused on a situation where the stronger states form a hegemonic-coalition and achieve positive security against any other coalitions that may form. Riker (1962) maintained that "minimum winning coalitions" continually form thereby rendering stability impossible. The opposing views can be divided into two broad categories: those indicating enforcement and managerial problems and those bringing in motives such as seeking absolute gains. The classical realist answer focuses on gains and points out that a winning coalition will give way to the creation of a global hegemon after division of gains and the weaker states will oppose this. Waltz (1979, pp. 168-169) draws the attention to managerial difficulties. Alliances among unequals is much easier to maintain than alliances among equals.[37]

[37] Morrow (1991) examines asymmetry among alliance members in detail.

Since winning coalitions usually require at least two members of nearly equal and relatively large capabilities, such an alliance will not endure simply because strong members are indispensable to make the coalition winning. A different explanation comes from the "bargaining type" game studied by Niou and Ordeshook (1986). The strong states will not be able to ally against the weak because the weak states, by offering better gains in an alternative coalition configuration or by voluntary transfer of resources, will be able to convince certain prospective members of the winning coalition against the idea.

Our characterization of system stability as a c-equilibrium suggests a less specific but at the same time more comprehensive behavior derived from the basic motive of security. We have shown (see fn. 34) that if a c-equilibrium, with the weakest state as a singleton exists, so does a c-equilibrium with any other state standing as a singleton. The following rule can hence be stated. *If no global hegemon exists and a hegemonic coalition of $n-1$ states is formed against any state-i, then state-i will act to break the grand coalition into at least two coalitions.* [38]

4 BALANCING IMPERATIVE

We have now a clear enough picture of a multipolar world to discuss the evasive (neo)realist notions "balance-of-power" and "balancing-act".

The results point out that a variety of notions of "equilibrium" exist each capturing a system status that is caused by a specific "motive" shared by states; motive of security causes a b-equilibrium or an approximate b-equilibrium, motive of certainty causes a search for a unique allocation profile, both motives combined result in a quest for a unique equilibrium. If one assumes, underrating the difficulties of forming and maintaining them, that alliances may form, then the proper notion of equilibrium is that of an a-equilibrium, or c-equilibrium at an extreme.

Our results distinguish between "a world in which equilibrium prevails" and "a world in which equilibrium is possible". The first requires an appropriate allocation profile whereas the second an appropriate resource distribution across the states. The distinction indicates two meanings attributable to "a balancing-act". It may first be an act towards "attaining an equilibrium when it exists" and it consists of a proper choice of allocations by each state. It may second be an act towards "making equilibrium possible" and can only involve a change in resource distribution; mainly through internal efforts by units. Note that (by our results of Section 3) "making alliances" can not be considered a balancing-act in this second sense since if an equilibrium does not exist, then there is a global hegemon which remains a hegemon under any alliance (from alignment to coalition) configuration. Breaking alliances can be a balancing act provided a hegemonic-coalition exists.

[38] Note that Niou and Ordeshook (1986) indicate how state-i will be able to achieve its aim by laying down specific rules in their model. As one would expect from rich models, some details stimulate arguments. Wagner (1993, p. 599n), for instance, finds preemptive resource transfer assumption problematic. Also see Niou, Ordeshook, and Rose (1989, p. 101n).

This distinction necessitates defining a "balanced world" as one in which a certain type of equilibrium *exists* whether or not this equilibrium is actually attained![39] The type of equilibrium may change depending on the nature of motives attributed to the states. Such a definition directly relates balance-of-power to capability distribution across the states.

Assumption	**Fixed Resource Distribution**				
World Status	Bilateral Equilibrium	Unique Alloc. Profile	Perfect Equilibrium	Alliance Equilibrium	Coalition Equilibrium
Motive	Security	Certainty	Security + Due emphasis of stronger	Security + Internal certainty in alliance	Security + Be hegemon
Act	Independent allocations	Unique allocations	Unique independent allocations	Internal and external allocations	Break any potential hegemonic coalition
Exists when	No hegemon	$n \leq 3$ or (near) hegemon or *other*	Two weakest states are not too weak	No global hegemon	No global hegemon

Table 1

The Tables 1 and 2 summarize our findings. In Table 1, it is assumed that the resource distribution is fixed. The top row indicates the system status. In rows 2 and 3, the motives that yield the status and the act that is geared towards attaining it are indicated. The last row shows under what resource distribution the respective status exists. In Table 2, we no longer assume that the resource distribution is fixed in the system, and list possible meanings that can be associated with "a balanced world". The last two columns constitute a crude summary of our interpretation of results from Niou, Ordeshook, and Rose (1989). The listed notions of "a balanced world" in Table 2 are possibilities that are open remaining inside the domain of structural theory.[40][41]

[39]This is in accordance with the position taken by many scholars. Zinnes (1971) points out that the meaning attributed to a balance of power in a majority of empirical or theoretical studies is the absence of a predominant country. Both meanings attributed by Niou and Ordeshook (1986) to balance, system stability and resource stability, are in terms of the distribution of resources. System stability as we have shown is equivalent to the existence of a particular c-equilibrium. Resource stability, on the other hand, is equivalent to the existence of a near global hegemon. Wagner (1986, 1994) in his attempts to define stability in a noncooperative context also focuses on resource distribution.

[40]Leaving the domain of structure theory many alternative notions are still possible. A balanced world can mean that there is a hierarchic order in the world and any "imbalance" (according to the definition of the highest in hierarchy) is kept in check. It can also mean that none of the major powers is hostile, they are all democracies, none is suffering from any internal turmoil. The first attributed meaning is obtained by dropping the assumption of anarchy and the second by deviating from the assumption that states are the units of the system.

[41]The last two columns are obtained only by suppressing the role of "enforcement problems" that coalitions will face. It is debatable whether this violates the self-help assumption of structural theory.

World Status	Balance	Perfect Balance	Resource Stability	System Stability
Motive	Security	Security + Due emphasis to stronger + Dynamic consideration (10)	No resource transfer	No state with zero resource
Act	Internal resource build-up	Internal resource build-up by weaker states	Create or become a near hegemon	Negotiate or act to break coalitions
Attained when	$\exists$ no hegemon	Two weakest states are not too weak	$\exists$ a near hegemon	The weakest state is not too weak

Table 2

The balancing imperative of Waltz (1979) can only be partly verified with the restricted model studied here. The claim that the balances of power tend to form given the structure and the motive of security is a dynamic claim while we have mainly studied static, snapshot models. The internal efforts and external efforts for establishing and maintaining a balance change either the number of effective units or the distribution of capabilities, or both. Hence, the balancing imperative is claimed to hold under changes in structure while holding the self-help nature of structure and the motive of security constant. The external efforts consist of forming or breaking alliances so that the claim contains alliances of all shades, not just alignments or coalitions we have concentrated on. A formal model which would verify the balancing imperative in its full generality would have to be dynamic, should incorporate a time dimension and acceptable "updating mechanisms" for allocations. Nonetheless, such limitations of our model simultaneously make its conclusions more universal: they apply to any point in time, to any resource distribution irrespective of how it resulted, etc. Theorem 1 and Proposition 1, summarized in the first columns of Tables 1 and 2, prove the following version of the balancing imperative:

Given a system of n states of fixed resources, where each state is independently maximizing its security measured as the minimum excess mutual allocation it can have against all other states, a world in as close a b-equilibrium as possible results. The b-equilibrium is exact if and only if there is no global hegemon. If there is a global hegemon, all other states, having negative securities against the hegemon, work towards increasing their resources via internal efforts.

According to our definition above, a world out-of-balance containing self-help units is a world containing a hegemon. In such a world, all states save the hegemon have to increase their capabilities through internal efforts in order to attain nonnegative securities. Once a balanced world emerges, security considerations indicate a behavior towards the attainment of a b-equilibrium. Concerning the process of reaching a b-equilibrium, we have established the following facts:

(i) Whenever there are more than one possible equilibria, the attainment of a b-equilibrium is a dynamic and uncertain process even under fixed resources.

(ii) As the gap between the strongest and the weakest state gets larger or as a hegemonic-alliance gets closer to being a coalition, increasingly larger and stronger alliances emerge from structural necessities.

(iii) If, in addition to maximizing their security, all states consistently and equally emphasize the stronger of any given pair of states by their allocations, then a world in as close a perfect equilibrium as possible results. The perfect equilibrium is exact and unique whenever the total resource of the two weakest states is not less than the average resource of the remaining states. The attainment of a perfect equilibrium is an instantaneous process and is devoid of uncertainty.

5 CONCLUSIONS

In no other field may a need for crystallization of ideas and notions be more pressing than in the field of international relations. Structural theory cleared some clouds but apparently not enough. Circular debates, misunderstandings, misrepresentations will not cease unless assertions and claims are supported by formal studies. Progress must come from clarity through formalization.

It is difficult to pinpoint which of the inferences obtained through the mathematical model studied here are already known and which are not; difficult, not because of a handicap in understanding what one reads, but because almost every imaginable international political proposition one may find already composed in the literature.

The notion of a b-equilibrium as equality of mutual allocations among every possible pairs of states has been central to all the results obtained. The novelty is in primarily focusing on allocations, apportionment of resources, even in the presence of alliances. Such a notion of equilibrium respects the multipolarity of the system at hand. The security of each state is defined in terms of the difference in its mutual allocations against all the other states. A strategic game of maximizing security characterizes the algebraic notion of b-equilibrium as a Nash equilibrium in the absence of a hegemon. The Nash equilibrium is an "approximate" b-equilibrium in the presence of a hegemon. The set of b-equilibria has been examined in detail in Özgüler, Güner, and Alemdar (1998). A b-equilibrium is unique if and only if $n \leq 3$ or there is a near hegemon. Whenever it is nonunique, there is an infinite number of equilibria and attaining a b-equilibrium becomes an uncertain and dynamic process due to a lack of coordination among the states. It is known only too well that the existence of a hegemon forbids and restrains, it is less known what its absence permits. A balancing imperative that follows by Theorem 1 and Proposition 1 is stated in Section 4. It may look like it is restating some very well known facts. Its content may look readily acceptable and trivial. It is, however, a statement in which every term is clearly defined and every assertion rigorously proved using notions key to structural realism.

In the simple security game we have considered, the action space is not restrictive enough to pin down a unique Nash equilibrium; arbitrarily large number of allocation profiles constitute equilibria. We then show, in Section 2.3, how similar ranking of any two states as to their likely potential for threat by a group of (or all) states would give further structure to equilibria, reducing the infinite set of equilibria possibly all the way down to one. Thus, we explicitly show how preferences interact with the structure (and *vice versa*) to shape the nature of equilibrium actions. It is observed and known that the states do not engage in endless shifts and adjustments of their allocations against other states, the allocations remain reasonably fixed unless resource distribution across the states changes. It is less known what considerations and motives might yield such a fixed allocation profile in a truly multipolar world.

Alliance formation is often indicated as the only remedy for a hegemon. One must be careful. A global hegemon remains a global hegemon no matter what alliances form. A *near* hegemon can be balanced by a grand coalition of all the other states and a coalition is the strongest form of an alliance. The states in a coalition are left with no possibility of an internal allocation when some internal conflict arises. The best remedy against a near hegemon is still an internal build-up of resources by some other states. In Section 3, we have considered alliances which at one extreme are alignments and at the other coalitions. We have illustrated using the notion of b-equilibrium and the concept of consistent equilibrium of Section 2.3, how similarity of views against third parties and motive of reducing uncertainty among a group of states can cause alliances. In Section 3.1, we have shown how structure and the motive of security cause endogenous formation of alliances of differing solidarity. There are two principal events which trigger endogenous alliance formation: first, one state approaches near global hegemony, second, a hegemonic-coalition emerges. We have examined coalition configurations from the point of view of equilibrium with particular emphasis on the weakest state. We have shown in Theorem 2, as common sense tells, that the more divided the world is, the easier it is for the weakest state to achieve security standing on its own feet. We have given a very different interpretation to the notion of system stability as the existence of a c-equilibrium in a three coalition configuration with the weakest state standing alone.

The simple notion of "equilibrium via allocations" is able to yield many *imaginable* characteristics of a multipolar world and, therefore, seems to be a very fundamental notion indeed.

6 APPENDIX

In proving Theorem 1, the terminology and notation of Osborne and Rubinstein (1994) will be used. The n-person strategic game considered consists of the set $\mathcal{N}$ of players, the set of actions $A_i = \{a_i \in \mathbf{R}^{n-1} : a_i = (a_{i1}, ..., a_{i(i-1)}, a_{i(i+1)}, ..., a_{in}),\ a_{ij} \geq 0,\ \sum_{j=1, j\neq i}^{n} a_{ij} = r_i\}$ available to player-i, and the utility function $u_i : \times_{i\in\mathcal{N}} A_i \rightarrow \mathbf{R}$ associated with player-i. By definition, a profile of actions $a \in \times_{i\in\mathcal{N}} A_i$ is a Nash equilibrium if

$$u_i(a) \geq u_i(a_{-i}, a_i') \ \forall\ i \in \mathcal{N},\ \forall\ a_i' \in A_i. \tag{16}$$

We first give explicit expressions for the *best response functions* of the states. Recall that the best response function of a player $i \in \mathcal{N}$ is $B_i(a_{-i}) = \{a_i \in A_i : u_i(a_{-i}, a_i) \geq u_i(a_{-i}, a_i')\ \forall\ a_i' \in A_i\}$. Given $a_i \in A_i$, let

$$a_{j_1 i} \geq a_{j_2 i} \geq ... \geq a_{j_{n-1} i},\ \{j_1, ..., j_{n-1}\} = \mathcal{N} - \{i\}$$

be an ordering of allocations against state-i. Define a *critical integer* $m(i) \in [2, n]$ as $m(i) = n$ if

$$\sum_{t=1}^{n-1} a_{j_t i} \leq r_i \tag{17}$$

and as the minimum integer $m \in \{2, ..., n\}$ satisfying

$$\frac{1}{m-1}(\sum_{t=1}^{m-1} a_{j_t i} - r_i) > a_{j_m i}, \tag{18}$$

where $a_{j_n i} = 0$, if (17) fails. Note that if (17) fails and $\frac{1}{n-1}(\sum_{t=1}^{n-1} a_{j_t i} - r_i) \leq a_{j_{n-1} i}$, then $m(i)$ is again equal to n. If (17) fails and $\frac{1}{n-1}(\sum_{t=1}^{n-1} a_{j_t i} - r_i) > a_{j_{n-1} i}$, then by the expression

$$\frac{1}{m-1}(\sum_{t=1}^{m-1} a_{j_t i} - r_i) - a_{j_m i} = \frac{m-2}{m-1}[\frac{1}{m-2}(\sum_{t=1}^{m-2} a_{j_t i} - r_i) - a_{j_{m-1} i}] + a_{j_{m-1} i} - a_{j_m i}$$

valid for $m > 2$, a minimum $m(i)$ exists and satisfies $m(i) \leq n - 1$.

Lemma 1. *The best response function of state-i is given by* $B_i(a_{-i}) = \{a_i \in A_i : a_{ij}$ satisfies (19) $\forall\ j \in \mathcal{N} - \{i\}\}$:

$$a_{ij_s} = \begin{cases} a_{j_s i} + \frac{1}{m(i)-1}(r_i - \sum_{t=1}^{m(i)-1} a_{j_t i}) \text{ for } s = 1, ..., m(i) - 1, \\ 0 \text{ for } s = m(i), ..., n-1 \text{ whenever } m(i) < n. \end{cases} \tag{19}$$

Proof. If $a_i = (a_{i1}, ..., a_{i(i-1)}, a_{i(i+1)}, ..., a_{in})$ is chosen by (19), then

$$a_{ij_s} - a_{j_si} = \begin{cases} \frac{1}{m(i)-1}(r_i - \sum\limits_{t=1}^{m(i)-1} a_{j_ti}) \text{ for } s = 1, ..., m(i)-1, \\ -a_{j_si} \text{ for } s = m(i), ..., n-1 \text{ whenever } m(i) < n, \end{cases}$$

so that, using (18), one has

$$\min_s\{a_{ij_s} - a_{j_si}\} = \frac{1}{m(i)-1}(r_i - \sum_{t=1}^{m(i)-1} a_{j_ti}) \tag{20}$$

for any $m(i) \in [2, n]$. Suppose, by way of contradiction, that some $a_i^* \in A_i$ achieves a better utility, i.e., $\min_s\{a^*_{ij_s} - a_{j_si}\} > \frac{1}{m(i)-1}(r_i - \sum_{t=1}^{m(i)-1} a_{j_ti})$. It follows that $a^*_{ij_s} > a_{j_si} + \frac{1}{m(i)-1}(r_i - \sum_{t=1}^{m(i)-1} a_{j_ti})$ for all $s = 1, ..., n-1$. Summing over $s = 1, ..., m(i)-1$, we obtain $\sum_{s=1}^{m(i)-1} a^*_{ij_s} > r_i$, which gives that $a_i^* \notin A_i$, a contradiction. This establishes that (19) gives the best response for any $i \in \mathcal{N}$. □

Here we note some properties of the best response function (19). By the definition of the critical integer and by (19) it holds for any $i \in \mathcal{N}$ that

$$m(i) = 2 \quad \Rightarrow \quad a_{ij_1} = r_i \text{ and } a_{ij_s} = 0, \ s \geq 2. \tag{21}$$

Further, if (17) holds, then $m(i) = n$ and either $r_i > \sum_{s=1}^{n-1} a_{j_si}$ in which case (19) gives that $a_{ij_s} - a_{j_si} > 0$ for all $s = 1, ..., n-1$ or $r_i = \sum_{s=1}^{n-1} a_{j_si}$ in which case $a_{ij_s} - a_{j_si} = 0$ for all $s = 1, ..., n-1$. If, on the other hand, (17) fails and $m(i) = n$, then (19) gives that $a_{ij_s} - a_{j_si} < 0$ for $s = 1, ..., n-1$. If (17) fails and $m(i) \leq n-1$, then $a_{ij_s} - a_{j_si} < 0$ for $s = 1, ..., m(i)-1$ and $a_{ij_s} - a_{j_si} \leq 0$ for $s = m(i), ..., n-1$. In particular, we have

$$\begin{array}{lll} a_{ij_s} > a_{j_si} \text{ for some } s \Rightarrow & a_{ij_s} > a_{j_si} \text{ for all } s, \\ a_{ij_s} = a_{j_si} \text{ for some } s \Rightarrow & \text{either } a_{ij_s} = a_{j_si} \text{ for all } s \text{ or } m(i) \leq n-1, \\ a_{ij_s} < a_{j_si} \text{ for some } s \Rightarrow & a_{ij_s} < a_{j_si} \text{ for } s < m(i) \text{ and } a_{ij_s} \leq a_{j_si} \text{ for } s \geq m(i). \end{array} \tag{22}$$

Proof of Theorem 1. *We first prove (i) under the assumption that there is a hegemon.* Let state-1 be a hegemon so that $r_1 > \sum_{i=2}^n r_i$. The profile of actions $a^0 := (a_1^0, ..., a_n^0) \in \times_{i \in \mathcal{N}} A_i$ which satisfies $a^0_{ij} = 0, \ \forall \ \{i,j\} \subset \mathcal{N} - \{1\}$, $a^0_{i1} = r_i, \ \forall \ i \in \mathcal{N} - \{1\}$, and $a^0_{1i} = r_1 + (r_1 - \sum_{j=2}^n r_j)/(n-1), \ \forall \ i \in \mathcal{N} - \{1\}$ yields $u_1(a^0) = (r_1 - \sum_{j=2}^n r_j)/(n-1)$ and $u_i(a^0) = -(r_1 - \sum_{j=2}^n r_j)/(n-1)$ for all $i \in \mathcal{N} - \{1\}$. We show that a^0 is a Nash equilibrium, i.e., $u_i(a^0) \geq u_i(a^0_{-i}, a_i)$, or equivalently, $0 \geq \min_{j \in \mathcal{N}-\{i\}}\{a_{ij} - a^0_{ji}\} \ \forall \ a_i \in A_i, \ \forall \ i \in \mathcal{N}$. If for some $k = 2, ..., n$ and some $a_k \in A_k$, $-(r_1 - \sum_{j=2}^n r_j)/(n-1) < \min_{j \in \mathcal{N}-\{k\}}\{a_{kj} - a^0_{jk}\}$, then $a_{kj} > a^0_{jk} - (r_1 - \sum_{j=2}^n r_j)/(n-1)$ for all $j \in \mathcal{N} - \{k\}$. This gives for $j = 1$ that $a_{k1} > r_k$ which is not possible. If on the other hand for some $a_1 \in A_1$, $(r_1 - \sum_{j=2}^n r_j)/(n-1) < \min_{j \in \mathcal{N}-\{k\}}\{a_{1j} - a^0_{j1}\}$, then $a_{1j} > a^0_{j1} + (r_1 - \sum_{j=2}^n r_j)/(n-1)$ for $j = 2, ..., n$ so that summation over j gives $r_1 > r_1$. This proves that a^0 is a Nash equilibrium. To prove that every Nash equilibrium is equal to a^0, we show that the best responses of Lemma 1 yield a^0. Since $r_1 > \sum_{j=2}^{n} r_j$, (17) holds with strict inequality and (19) gives

$$a_{1i} = a_{i1} + \frac{1}{n-1}(r_1 - \sum_{j=2}^{n} a_{j1}),\ i = 2, ..., n,$$

with $a_{1i} > a_{i1}$ for $i = 2, ..., n$. Since every state $k = 2, ..., n$ has negative security against state-1, by (22), it follows that $a_{ki} \leq a_{ik}$ and that the critical integers satisfy $m(k) < n$ for all $\{k, i\} \subset \mathcal{N} - \{1\}$. This implies in particular that $a_{ki} = a_{ik}$ for all $\{k, i\} \subset \mathcal{N} - \{1\}$. Consider any $k \in \mathcal{N} - \{1\}$ and let $a_{j_1 k} \geq .. \geq a_{j_{n-1} k}$ be the allocations against state-k with the critical integer $m(k) < n$. If $m(k) > 2$, then since by (19) $a_{k1} - a_{1k} = \frac{1}{m(k)-2} \sum_{t=1} m(k) - 1(a_{j_t k} - a_{k j_t})$, it follows that $a_{1k} = a_{k1}$ which does not hold. Hence, $m(k) = 2$ and by (21), $a_{k1} = r_k$ for $k = 2, ..., n$, and $a_{kj} = 0$ for $\{k, j\} \subset \mathcal{N} - \{1\}$. This proves (i) in case there is a hegemon.

We now prove (i) under the assumption that there is no hegemon. By Proposition 1, a b-equilibrium is a profile of actions $a^0 := (a_1^0, ..., a_n^0) \in \times_{i \in \mathcal{N}} A_i$ which satisfies $a_{ij}^0 = a_{ji}^0,\ \forall\ i \neq j$ and yields $u_i(a^0) = 0$ for all $i \in \mathcal{N}$. We show that a b-equilibrium is a Nash equilibrium, i.e., $0 \geq \min_{j \in \mathcal{N} - \{i\}} \{a_{ij} - a_{ji}^0\}\ \forall\ a_i \in A_i,\ \forall\ i \in \mathcal{N}$. If for some $k \in \mathcal{N}$ and some $a_k \in A_k$, $0 < \min_{j \in \mathcal{N} - \{k\}} \{a_{kj} - a_{jk}^0\}$, then $a_{kj} > a_{jk}^0$ for all $j \in \mathcal{N} - \{k\}$ so that $r_k = \sum_{j \in \mathcal{N} - \{k\}} a_{kj} > \sum_{j \in \mathcal{N} - \{k\}} a_{jk}^0 = \sum_{j \in \mathcal{N} - \{k\}} a_{kj}^0 = r_k$ giving a contradiction. This proves that a^0 is a Nash equilibrium.

Next, we need to show that every Nash equilibrium is a b-equilibrium, i.e., if a profile a satisfies (16), and hence (19) of Lemma 1, then $a_{ij} = a_{ji},\ \forall\ i, j \in \mathcal{N}, i \neq j$. Suppose that for two arbitrary states, say state-1 and state-2, one has $a_{12} \neq a_{21}$. Let $a_{j_1 1} \geq ... \geq a_{j_{m(1)-1} 1} \geq a_{j_{m(1)} 1} \geq .. \geq a_{j_{n-1} 1}$ be the allocations against state-1 with the critical integer $m(1)$. Let $k \in \{1, ..., n-1\}$ be such that $j_k = 2$. Also let $a_{i_1 2} \geq ... \geq a_{i_{m(2)-1} 2} \geq a_{i_{m(2)} 2} \geq .. \geq a_{i_{n-1} 2}$ be the allocations against state-2 with the critical integer $m(2)$. Let $l \in \{1, ..., n-1\}$ be such that $i_l = 1$. Since $a_{12} \neq a_{21}$, one of the following three cases occur: *Case 1.* $k < m(1)$ and $l \geq m(2)$ which implies by (19) that $a_{21} = 0$, $a_{12} > 0$, and $a_{12} = a_{i_l 2} = \frac{1}{m(2)-1} (\sum_{t=1}^{m(2)-1} a_{i_t 2} - r_2)$. Thus, expressing r_2 as the sum of allocations of state-2,

$$\sum_{t=1}^{m(2)-1} (a_{2 i_t} - a_{i_t 2}) < - \sum_{t=m(2)}^{n-1} a_{2 i_t} \leq 0.$$

Thus, the sum on the left hand side contains a nonzero term. Since $l \geq m(2)$, it must be that

$$a_{ij} \neq a_{ji} \text{ for some } i > 1, j > 1, i \neq j. \tag{23}$$

Case 2. $k \geq m(1)$ and $l < m(2)$ which implies that $a_{12} = 0$ and $a_{21} > 0$. Thus, $a_{21} - a_{12} > 0$. By (22), $a_{2j} - a_{j2} > 0$ for all $j = 1, 3, ..., n$ so that (23) holds. *Case 3.*

$k < m(1)$ and $l < m(2)$ which implies by (19) that $a_{21} - a_{12} = \frac{1}{m(2)-1}(\sum_{t=1}^{m(2)-1} a_{i_t 2} - r_2)$. Since $a_{2i_s} = 0$ for $s = m(2), ..., n-1$, we can write $r_2 = \sum_{t=1}^{m(2)-1} a_{2i_t}$ so that $a_{21} - a_{12} = \sum_{t=1}^{m(2)-1}(a_{2i_t} - a_{i_t 2})$. Since $l < m(2)$, the term $a_{21} - a_{12}$ also appears in the sum if $m(2) > 2$. Hence, this equality gives $a_{21} - a_{12} = \sum_{t=1}^{m(2)-2}(a_{2i_t} - a_{i_t 2})$ provided $m(2) > 2$. It follows, as $a_{21} - a_{12} \neq 0$ that (23) again holds when $m(2) > 2$. If $m(2) = 2$, then $l = 1$, $a_{12} - r_2 > a_{i_2 2}$ and(19) gives that $a_{21} = r_2$, $a_{2j} = 0$, $j = 3, ..., n$. Moreover, by $k < m(1)$ and by (19), $a_{12} = r_2 + \frac{1}{m(1)-1}(r_1 - \sum_{t=1}^{m(1)-1} a_{j_t 1})$. Since $a_{12} - a_{21} = a_{12} - r_2 > 0$, it follows that $r_1 > \sum_{t=1}^{m(1)-1} a_{j_t 1}$ which implies that (18) fails. It must then be that $m(1) = n$ and

$$a_{1i} = a_{i1} + \frac{1}{n-1}(r_1 - \sum_{j \in \mathcal{N}-\{i\}} a_{j1}), \ j = 2, ..., n. \tag{24}$$

Let us consider any $j \geq 3$ and let $a_{q_1 j} \geq ...a_{q_{m(j)-1} j} \geq a_{q_{m(j)} j} \geq ...a_{q_{n-1} j}$ be the allocations against state-j with the critical integer $m(j)$. Let $x = 1, ..., n-1$ be that integer for which $q_x = 1$. Note that by (24), we have $a_{1j} - a_{j1} > 0$. If $x \geq m(j)$, then $a_{j1} = a_{jq_x} = 0$ and $a_{1j} = a_{q_x j} < \frac{1}{m(j)-1}(\sum_{t=1}^{m(j)-1} a_{q_t j} - r_j)$ implying by a similar argument to Case 1 that (23) holds. If $x < m(j)$, then since $k < m(1)$ we consider the cases $m(j) > 2$ and $m(j) = 2$ and conclude, similar to above, that either (23) holds or $a_{j1} = r_j$. Since $j \geq 3$ was arbitrary, either (23) holds or $a_{j1} = r_j$ for $j = 2, ..., n$. The second possibility, however, is prohibited since it would imply by $r_1 > \sum_{j=2}^{n} a_{j1}$ that the state-1 is a hegemon. Therefore, (23) also holds in Case 3. We have so far established that *if* $(a_{ij})_{\{i,j\} \subset \mathcal{N}}$ *is a Nash equilibrium and* $a_{12} \neq a_{21}$, *then* $a_{ij} \neq a_{ji}$ *for some* $\{i,j\} \subset \mathcal{N} - \{1\}$. We now show that, *if* $(a_{ij})_{\{i,j\} \subset \mathcal{N}}$ *is a Nash equilibrium, then*

$$r_i - a_{i1} \leq \sum_{j \in \mathcal{N}-\{1,i\}} (r_j - a_{j1}), \ \forall \ i \in \mathcal{N} - \{1\}, \tag{25}$$

i.e., there is no hegemon among the $n-1$ *states of resources* $r_2 - a_{21}, ..., r_n - a_{n1}$. Suppose, by way of contradiction, that (25) fails for some $i \in \mathcal{N} - \{1\}$, say $i = 2$. Then, $r_2 - a_{21} > \sum_{j=3}^{n}(r_j - a_{j1})$ and (19) gives that

$$a_{2j} = r_j - a_{j1} + \frac{1}{n-2}[r_2 - a_{21} - \sum_{j=3}^{n}(r_j - a_{j1})], \ j = 3, ..., n,$$

and $a_{kj} = 0$ for $\{k, j\} \in \mathcal{N} - \{1\}$. Thus, state-2 has positive security against states $3, ..., n$ and, by (22), also against state-1. State-1 having negative security against state-2 should have nonpositive security against all other states, by (22), so that $a_{j1} \geq a_{1j}$ for all $j = 3, ..., n$. If $a_{k1} > a_{1k}$ for some $k \in \{3, ..., n\}$, then, again by (22), state-k should have positive security against state-2, which is not the case. Hence, $a_{j1} = a_{1j}$ for all $j = 3, ..., n$. It follows, by (22), that either $r_1 = \sum_{j=2}^{n} a_{j1}$ or $m(1) = 2$. However, the first possibility would imply $a_{12} = a_{21}$ so that $m(1) = 2$. Therefore, $a_{1j} = 0$ for $j = 3, ..., n$ and $a_{12} = r_1$. This necessitates $a_{j1} = 0$ for $j = 1, ..., n$ and $a_{j2} = r_j$ for $j = 1, 3, ..., n$.

Therefore, $r_2 > r_1 + ... + r_n$ contradicting our assumption that there is no hegemon. This proves that there is no hegemon among the states of resources $r_j - a_{j1},\ j = 2, ..., n$ as claimed. By induction, there should be no hegemon among some two states k, l of resources $r_k - \sum_{t\in\mathcal{N}-\{k,l\}} a_{kt}$ and $r_l - \sum_{t\in\mathcal{N}-\{k,l\}} a_{lt}$, whereas, $a_{kl} \neq a_{lk}$. Since this is not possible, we must have $a_{ij} = a_{ji}$ for all $\{i, j\} \subset \mathcal{N}$ contrary to our initial assumption. This proves (i) for the case of no hegemon.

The statement (ii) follows by (i) and by the Proposition 3 of Özgüler, Güner, and Alemdar (1998) on noting by

$$\max_{a\in A} \min\{u_1(a), ..., u_n(a)\} = -\min_{a_{ij}} \max_{i,j,\, i\neq j} |a_{ij} - a_{ji}| \tag{26}$$

that the optimization problem in (ii) is equivalent to minimizing $\max_{i,j,\, i\neq j} |a_{ij} - a_{ji}|$ over all $a \in A$.

We now prove (iii). By definition, a profile $a \in A$ is strongly Pareto efficient if there is no $a' \in A$ for which $u_i(a') \geq u_i(a)$ for all $i \in \mathcal{N}$ with strict inequality for at least one $i \in \mathcal{N}$. By (ii), a Nash equilibrium a^* maximizes $\min_i\{u_i(a)\}$ so that

$$\min_i\{u_i(a)\} \leq \min_i\{u_i(a^*)\},\ \forall\, a \in A. \tag{27}$$

Suppose, for some $a' \in A$, $u_i(a') \geq u_i(a*)$ for all $i = 1, ..., n$ with strict inequality for some $k = 1, ..., n$. Then, $\min_i\{u_i(a')\} \geq \min_i\{u_i(a*)\}$ which implies, as (27) also applies to a', that $\min_i\{u_i(a')\} = \min_i\{u_i(a*)\}$. Hence, a' also maximizes (6) and is also a Nash equilibrium profile by (ii). It follows that $u_i(a') = u_i(a*)$ for all i and a strict inequality is not possible. □

Proof of Proposition 3. It is clear that perfect equilibrium satisfy (10) by the expressions $a_{ij} = \frac{1}{n-1}\left(r_i + r_j - \frac{1}{n-2}\sum_{t\in\mathcal{N}-\{i,j\}} r_t\right)\ \ \forall\ \ \{i, j\} \subset \mathcal{N}$ of allocations. We prove the converse. Let $a_{ij} = f_{ij}(r_1, ..., r_n)$ be continuous and differentiable with respect to each $r_l, l \in \mathcal{N}$. If a_{ij}'s constitute a b-equilibrium, then $\lim_{r_k\to 0\,\forall k} f_{ij}(r_1, ..., r_n) = 0$ for each i, j. Hence, to show that (10) implies perfect equilibrium, it is enough to establish that (10) implies

$$\frac{\partial a_{ij}}{\partial r_l} = \begin{cases} \frac{1}{n-1} & \text{if } l \in \{i,j\}, \\ -\frac{1}{(n-1)(n-2)} & \text{if } l \notin \{i,j\}, \end{cases} \tag{28}$$

i.e., the derivatives are constant with the values shown. Note that for an arbitrary $i \in \mathcal{N}$, say $i = 1$, we have $\sum_{j=2}^{n} a_{1j} = r_1$ which gives by differentiating with respect to r_1 and employing (10) with $l = 1$ that $(n-1)\frac{\partial a_{1j}}{\partial r_1} = 1$ for any $j = 2, ..., n$. Hence, (28) holds for $l \in \{i, j\}$. On the other hand, $a_{j1} + \sum_{t\in\mathcal{N}-\{1,j\}} a_{jt} = r_j$ for $j = 2, ..., n$. Using $a_{j1} = a_{1j}$, differentiating with respect to r_1, and employing (10), we obtain $(n-2)\frac{\partial a_{jt}}{\partial r_1} = -\frac{1}{n-1}$ for any $\{j, t\} \in \mathcal{N} - \{1\}$. Hence, (28) also holds for $l \notin \{i, j\}$. □

Proof of Theorem 2. (i) It is clear that the existence of a c-equilibrium implies that there is no global hegemon. We prove the converse. Let k be the smallest integer such that the following inequalities are satisfied:

$$\sum_{i=1}^{k} r_{2i-1} \leq \sum_{j=1}^{n} r_j - \sum_{i=1}^{k} r_{2i-1},$$

$$\sum_{i=1}^{k+1} r_{2i-1} \geq \sum_{j=1}^{n} r_j - \sum_{i=1}^{k+1} r_{2i-1}.$$

Thus k is such that the sum of the resources of the odd-numbered states $1, 3, ..., 2k-1$ is not more than the sum of the resources of the remaining states but when the resource of the state $2k+1$ is added, the situation is reversed. By the absence of a hegemon, such a $k \geq 1$ always exists and by the ordering (14) it satisfies $2k+1 \leq n$ for $n \geq 3$. Consider the coalitions

$$\mathcal{C}_1 = \{1, 3, ..., 2k-1\},\ \mathcal{C}_2 = \{2, 4, ..., 2k\},\ \mathcal{C}_3 = \{2k+1, ..., n-1\},$$

where $\mathcal{C}_3$ is the empty coalition if $n = 2k+1$ and is then dropped. By the choice of k, we have

$$r(\mathcal{C}_1) \leq r(\mathcal{C}_2) + r(\mathcal{C}_3) + r_n, \tag{29}$$

$$r(\mathcal{C}_1) + r_{2k+1} \geq r(\mathcal{C}_2) + r(\mathcal{C}_3) - r_{2k+1} + r_n. \tag{30}$$

Hence, by (14) and (29), we have $r(\mathcal{C}_1) \leq r(\mathcal{C}_2) \leq r(\mathcal{C}_2) + r(\mathcal{C}_3) + r_n$. Also, by (30), $r(\mathcal{C}_3) \leq r(\mathcal{C}_1) + r(\mathcal{C}_2) + r_n - 2[r(\mathcal{C}_2) + r_n - r_{2k+1}]$. Since $r_{2k} \geq r_{2k+1}$, the term in brackets is positive so that $r(\mathcal{C}_3) \leq r(\mathcal{C}_1) + r(\mathcal{C}_2) + r_n$. Consequently, with coalitions as defined above there is no hegemonic-coalition and a c-equilibrium with $l \leq 4$ is possible. Note that the above costruction gives $l = 3$ if $\mathcal{C}_3$ is the empty coalition and $l = 4$ if it is nonempty. Given now any $4 < l \leq n$, by partitioning $\mathcal{C}_1, \mathcal{C}_2, \mathcal{C}_3$ *separately* to obtain $l-1$ coalitions, no hegemonic-coalition is created and a c-equilibrium of l coalitions can be obtained. (ii) The proof is immediate on noting that the claimed existence condition is

$$r_n \geq \min_{\mathcal{C}_1, \mathcal{C}_2} |r(\mathcal{C}_1) - r(\mathcal{C}_2)|,\ \mathcal{C}_1 \cup \mathcal{C}_2 = \{1, ..., n-1\},\ \mathcal{C}_1 \cap \mathcal{C}_2 = \emptyset,$$

where the minimum always exists. □

REFERENCES

Bueno de Mesquita, B. and D. Lalman. 1992. *Reason and War*, New Haven: Yale University Press.

Christensen, T.J. and J. Snyder. 1990. "Chain gangs and passed bucks: Predicting alliance patterns in multipolarity." *International Organization.* 44(Spring): 137-168.

Chvátal, V. 1983. *Linear Programming*, W. H. Freeman and Company, New York.

Deutsch, K. W. and J. D. Singer. 1964. "Multipolar power system and international stability." *World Politics.* 16(April):390-406.

Gilpin, R. 1981. *War and Change in World Politics.* Cambridge: Cambridge University Press.

Grieco, J. 1990. *Cooperation Among Nations: Europe, America, and Non-tariff Barriers to Trade.* Ithaca: Cornell University Press.

Grieco, J., R. Powell, and D. Snidal. 1993. "The relative-gains problem for international cooperation." *American Political Science Review.* 93(September): 729-743.

Jervis, R. 1978. "Cooperation under security dilemma." *World Politics.* 30: 167-214.

Legro, J. W. and A. Moravcsik. 1999. "Is anybody still a realist?" *International Security.* 24/2(Fall). pp. 5-55.

Morrow, J. D. 1991. "Alliances and asymmetry: An alternative to the capability aggregation model of alliances." *American Journal of Political Science* 35(4): 904-933.

Niou, E. M. S. and P. C. Ordeshook. 1986. "A theory of balance of power in international systems." *Journal of Conflict Resolution* 30(4): 685-715.

Niou, E. M. S. and P. C. Ordeshook. 1989. "The geographical imperatives of the balance of power in 3-country systems." *Mathematical and Computer Modelling* 12(4/5): 519-31.

Niou, E. M. S., P. C. Ordeshook, and G. F. Rose. 1989. *The Balance of Power: Stability in International Systems,* Cambridge University Press, New York.

Olson, M. and R. Zeckhauser. 1966. "An economic theory of alliance." *The Review of Economics and Statistics.* 48: 266-79.

Osborne, M. J. and A. Rubinstein. 1994 *A Course in Game Theory,* The MIT Press, Cambridge, Massachusetts.

Özgüler, A. B., S. Ş. Güner, and N. M. Alemdar. 1998. "Balances of power from static equilibria." *Preprint.* Bilkent University, Ankara, Turkey.

Özgüler, A. B., S. Ş. Güner, and N. M. Alemdar. 1999. "Interval constrained equilibrium." *Preprint.* Bilkent University, Ankara, Turkey.

Powell, R. 1991. "Absolute and relative gains in international relations theory." *American Political Science Review.* 85: 1303-20.

Powell, R. 1993. "Guns, butter, and anarchy." *American Political Science Review.* 87(March): 115-32.

Powell, R. 1994. "Anarchy in international relations theory: the neorealist-neoliberal debate." *International Organization.* 48(Spring): 313-344.

Riker, W. H. 1962. *The Theory of Political Coalitions.* New Haven, CT: Yale University Press.

Russett, B. and H. Starr. 1992. *World Politics: The Menu for Choice.* New York: W. H. Freeman.

Schweller, R. L. 1994. "Bandwagoning for profit: Bringing the revisionist state back in." *International Security.* 19(Summer): 72-107.

Schroeder, P. 1976. "Alliances, 1815-1945: Weapons of power and tolls of management." In *Historical Dimensions of National Security Problems.* Klaus Knorr (ed.), Lawrence: University Press of Kansas.

Snyder, H. G. 1990. "Alliance theory: a neorealist first cut." *Journal of International Affairs.* 44: 103-24.

Van Evera, S. 1998. "Offense, defense, and the causes of war." *International Security.* 22: 5-43.

Wagner, R. H. 1986. "The theory of games and the balance of power." *World Politics* 38(4): 546-76.

Wagner, R. H. 1994. "Peace, war, and the balance of power." *American Political Science Review* 88(September): 593-607.

Walt, S. M. 1987. *The Origins of Alliances.* Ithaca, NY: Cornell University Press.

Waltz, K. 1979. *Theory of International Politics.* Reading, Mass.: Addison-Wesley.

Waltz, K. 1993. "The emerging structure of international politics." *International Security.* 18(Fall):44-79.

Waltz, K. 1997. "Evaluating theories." *American Political Science Review.* 91(December): 913-917.

Wendt, A. 1992. "Anarchy is what states make of it: the social construction of power politics." *International Organization.* 46: 391-425.

Zakaria, F. 1998. *From Wealth to Power: The Unususal Origins of America's World Role.* Princeton: Princeton University Press.

Zinnes, D. 1970. "Coalition theories and the balance of power," in S. Groennings et al. (eds.) *The Study of Coalition Behavior.*, New York: Holt, Rinehart, and Winston.