1. Given

\[
A = \begin{bmatrix}
-1 & 1 & 0 & 1 & 0 \\
0 & -3 & 0 & -2 & 0 \\
-2 & 0 & -1 & 1 & 0 \\
0 & 2 & 0 & 1 & 0 \\
0 & 2 & 0 & 2 & -1
\end{bmatrix}
\]

(a) Find the characteristic and minimal polynomials, a modal matrix and the Jordan form of \( A \).

(b) Find the solution of \( \dot{x} = Ax \), corresponding to \( x_0 = \begin{bmatrix} 0 & 1 & 0 & 0 & -1 \end{bmatrix}^T \) and decompose into its modes.

2. Consider the system

\[
\begin{align*}
\dot{x}_1(t) &= (\cos t)x_2(t) \\
\dot{x}_2(t) &= - (\cos t)x_1(t) + u(t) \\
y(t) &= x_1(t)
\end{align*}
\]

(a) Obtain the state transition matrix.

(b) Obtain the impulse response

(c) Find \( \rho(t, 0, \theta, \cos t) \) for \( t \geq 0 \).

3. Show that for a linear, time-invariant system with an impulse response \( h(t) \),

\[
\int_{-\infty}^{\infty} y(t)dt = \int_{-\infty}^{\infty} h(t)dt \int_{-\infty}^{\infty} u(t)dt
\]

4. Describe the dual of the system in Q.2, find the state transition matrix, and verify the relation

\[
x^*(t)\bar{x}(t) + \int_{t_0}^{t} y^*(\tau)\bar{u}(\tau)d\tau = x^*(t_0)\bar{x}(t_0) + \int_{t_0}^{t} u(\tau)\bar{y}(\tau)d\tau
\]

for arbitrary initial states \( x(t_0), \bar{x}(t_0) \) and the inputs \( u(t) = \bar{u}(t) = \cos t \).
5. Define the dual of the discrete–time system

\[
\begin{align*}
x(k + 1) &= A(k)x(k) + B(k)u(k) \\
y(k) &= C(k)x(k) + D(k)u(k)
\end{align*}
\]

and express the state–transition matrix of the dual system in terms of that of the original.

6. Show that if \( A(t) \) and \( A(\tau) \) commute for all \( t, \tau \in \mathbb{R} \), then the state transition matrix of the linear differential equation \( \dot{x}(t) = A(t)x(t) \) is

\[
\Phi(t, \tau) = e^{\int_{t}^{\tau} A(\sigma) d\sigma}.
\]

7. In the following figures the systems \( S_1 \) and \( S_2 \) are described as

\[
\begin{align*}
S_1: \quad \dot{x}_1(t) &= (\cos t)x_1(t) + u_1(t) \\
y_1(t) &= (\cos t)x_1(t)
\end{align*}
\]

\[
\begin{align*}
S_2: \quad \dot{x}_2(t) &= u_2(t) \\
y_2(t) &= (\cos t)x_2(t)
\end{align*}
\]

Obtain the impulse response of each of the interconnected systems.

8. In the following feedback configuration \( S_1 \) and \( S_2 \) are identical systems described by \( \ddot{y}_i(t) = a(t)y_i(t) \), where \( a(.) \) is an integrable function. Obtain the impulse response of the system.