Discuss solution properties of the periodically varying discrete-time system

\[
\begin{align*}
\mathbf{x}[k+1] &= A[k]\mathbf{x}[k] + B[k]\mathbf{u}[k], \quad \mathbf{x}[k_0] = \mathbf{x}_0 \\
y[k] &= C[k]\mathbf{x}[k] + D[k]\mathbf{u}[k]
\end{align*}
\]

where the coefficient matrices \(A, B, C, D\) and the input \(\mathbf{u}\) are periodic with period \(K\):

\[
A[k+K] = A[k], \ldots, D[k+K] = D[k] \quad \text{and} \quad \mathbf{u}[k+K] = \mathbf{u}[k] \quad \text{for all} \quad k \in \mathbb{Z}
\]

1. Obtain an expression for the state transition matrix, which can be used to compute \(\Phi[k, k_0]\) for all \(k\) using only its values for \(k_0 \leq k < k_0 + K\).

2. Obtain necessary and/or sufficient conditions for the system to have periodic solutions under zero and also nonzero (periodic) inputs.

3. Illustrate your results with a simple (second order) but nontrivial example.