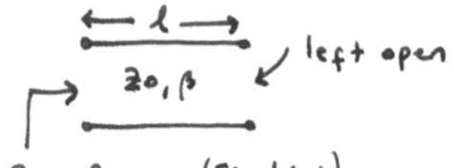
EEE 451 / 551

HW # 4

① Consider a lossless, electrically very short transmission line (i.e., $\beta l \ll 1$).

(a) Show that if the right end of this transmission line is left open (see Fig. 1(a)), it behaves like a capacitor. For this case, find its input impedance in terms of ω, L, C, l.

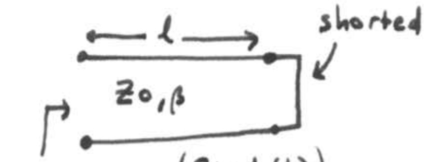


$Z_{in} = ?$ (Fig. 1(a))

L: inductance per unit length

C: capacitance per unit length.

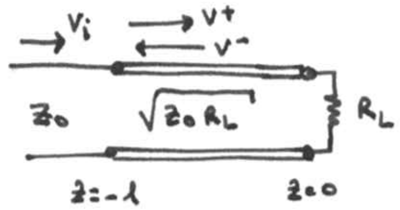
(b) Show that if the right end of this transmission line is shorted (see Fig. 1(b)), it behaves like an inductor. For this case, find its input impedance in terms of ω, L, C, l.



$Z_{in} = ?$ (Fig 1(b))

② Consider a section of a transmission line with length l. The output of this transmission line is left open and its input impedance is measured as Z_{in}^{oc}. Similarly its output is shorted and its input impedance is measured as Z_{in}^{sc}. Using these two measurements, determine its characteristic impedance Z_0 and the propagation constant β in terms of Z_{in}^{oc}, Z_{in}^{sc}, l. Assume the transmission line is lossless.

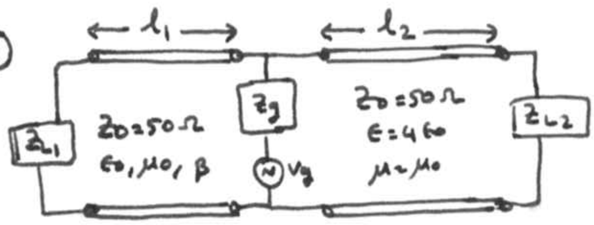
③



(Fig. 2)

Consider the quarter-wave matching transformer shown in Fig. 2. Derive expressions for V^+ and V^-, the amplitudes of the forward and reverse travelling waves on a quarter-wave line section, in terms of V^i, the incident voltage amplitude. The line is lossless.

④

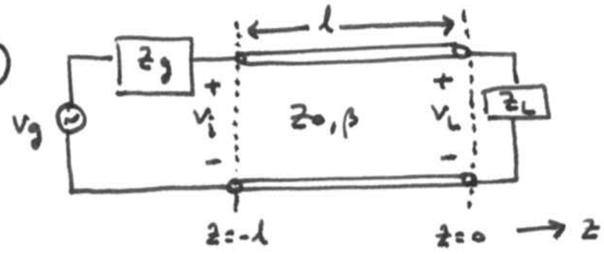


(Fig. 3)

A 12^V ($V_g = 12^V$) and 50Ω ($Z_g = 50\Omega$) generator is connected to a pair of lossless 50Ω transmission lines each terminated in a real load Z_{Li} where $Z_{Li} > Z_0$ is known for $i = 1, 2$. The transmission line circuit is shown in Fig. 3. The operating frequency is 1 GHz and both lengths, l_1 and l_2, are selected to be 7.5 cm. The ϵ and μ values for each transmission line are given on the →

figure. If VSWR (SWR or S) is measured to be 2 in both ends, calculate the power dissipated in each load resistor.

⑤



(Fig. 4)

A 100 MHz generator with $V_g = 10\angle 0°$ volts and internal resistance 50Ω is connected to a lossless 50Ω transmission line with ϵ_0, μ_0. The line is 3.6 m. long and it is terminated in a $25+j25\Omega$ load as seen in Fig. 4. Find the followings:

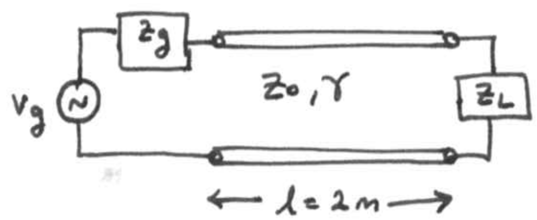
(a) The voltage at the input terminals, V_i.

(b) The voltage at the load end, V_L.

(c) The VSWR on the line.

(d) The average power delivered to the load.

⑥



(Fig. 5)

The transmission line circuit shown in Fig. 5 has the following parameters: $V_g = 10^V$, $z_g = 100\Omega$, $Z_0 = 100\Omega$, $Z_L = 200\Omega$, $f = 1\,GHz$, $\ell = 2m$ and $\alpha = 1.5\,dB/m$.

(a) Calculate the input and output (i.e., load) power.

(b) Calculate the return loss at the input and at the load end.

⑦ In class we expressed V_0^+ as $V_0^+ = V_g \dfrac{z_{in}}{z_{in}+z_g} \dfrac{1}{(e^{j\beta\ell} + \Gamma_L e^{-j\beta\ell})}$ when both z_g (generator impedance) and z_L (load impedance) are different than z_0 (characteristic impedance). Then recognizing that $\Gamma_g = \dfrac{z_g - z_0}{z_g + z_0}$, V_0^+ is expressed as

$$V_0^+ = V_g \frac{z_0}{z_0 + z_g} \frac{e^{-j\beta\ell}}{(1 - \Gamma_L \Gamma_g e^{-j2\beta\ell})} \quad \text{——} (*) .$$

In this question derive (*).