EEE 451 / 551 HW#7

(1) Show that for a parallel plate waveguide, the wave impedances for TM and TE modes are given by

$$Z_{TM} = \eta \sqrt{1-(f_c/f)^2}\ \Omega \quad \text{and} \quad Z_{TE} = \eta \Big/ \sqrt{1-(f_c/f)^2}, \text{ respectively.}$$

Note that $\eta = \sqrt{\mu/\epsilon}$ and f_c is the cut-off frequency.

(2) A TE_{10} wave at 10 GHz propagates in a brass rectangular waveguide with inner dimensions $a = 1.5$ cm, $b = 0.6$ cm, which is filled with polyethylene ($\epsilon_r = 2.25$, $\mu_r = 1$). Determine
- (a) the phase constant (k_z)
- (b) the guide wavelength (λ_g)
- (c) the phase velocity (v_p) along the guide
- (d) the wave impedance

(3) Standard air-filled waveguides have been designed for different radar bands. One type, designated WG-16, is suitable for X-band applications. Its dimensions are $a = 2.29$ cm and $b = 1.02$ cm. If it is desired that a WG-16 waveguide only in the dominant TE_{10} mode and that the operating frequency be at least 25% above the cut-off frequency of the TE_{10} mode but no higher than 95% of the next higher cut-off frequency, what is the allowable operating frequency range?

(4) A 10 GHz signal is to be transmitted inside a hallow circular conducting pipe. Determine the inside diameter of the pipe such that its lowest cut-off frequency is 20% below this signal frequency. If the pipe is to operate at 15 GHz, what waveguide modes can propagate in the pipe?

Zeros of $J_n(x) = p_{nm}$

n	p_{n1}	p_{n2}
0	2.405	5.520
1	3.832	7.016
2	5.136	8.417

Zeros of $J_n'(x) = p'_{nm}$

n	p'_{n1}	p'_{n2}
0	3.832	7.016
1	1.841	5.331
2	3.054	6.706

⇒

(5) Another method of obtaining TE, TM and TEM waves in guiding structures is a 2 step procedure where we make use of the vector potentials $\bar{A}$ and $\bar{F}$. Similar to $\bar{A}$, $\bar{F}$ arises from the solenoidal nature of $\bar{D}$ in a source free region (i.e. $\nabla\cdot\bar{D}=0 \Rightarrow \bar{D}=-\nabla\times\bar{F}$). In this 2 step procedure, the first step is to find the vector potentials $\bar{A}$ and $\bar{F}$ and then the second step is to find the fields from the determined potentials. Note that in many problems (involving the excitation of the waveguide), this procedure is simpler than the direct solution of the fields from the sources.

(a) Write down Maxwell's equations for a simple, lossless, source free region $\left(e^{j\omega t}\right)$

(b) Show that $\bar{H}_F = -j\omega\bar{F} - \frac{j}{\omega\mu\epsilon}\nabla(\nabla\cdot\bar{F})$ where $\bar{H}_F$ is the magnetic field intensity due to $\bar{F}$ potential. Use a scalar magnetic potential Φ_m when necessary, and obtain your own gauge for $\nabla\cdot\bar{F}$.

(c) Similar to part (b), show that $\bar{E}_A = -j\omega\bar{A} - \frac{j}{\omega\mu\epsilon}\nabla(\nabla\cdot\bar{A})$ where $\bar{E}_A$ is the electric field intensity due to $\bar{A}$ potential.

* Note that the final fields $\bar{E}$ and $\bar{H}$ are the superposition of $\bar{E}$ and $\bar{H}$ fields due to both $\bar{A}$ and $\bar{F}$. In other words, $\bar{E} = \bar{E}_A + \bar{E}_F$, $\bar{H} = \bar{H}_A + \bar{H}_F$

(d) Let $\bar{F} = \hat{a}_z \Psi_f(x,y,z)$ and $\bar{A}=0$ where $\Psi_f(x,y,z)$ is a known scalar quantity. Derive expressions for $E_x, E_y, E_z, H_x, H_y, H_z$ in terms of $\Psi_f(x,y,z)$. Determine if these expressions are valid for TE or TM waves. Also clearly show that $k=k_z$ for TEM waves. (Hint: z-variation is in the form of $e^{\pm jk_z z}$)

(e) Repeat part (d) for $\bar{A} = \hat{a}_z\Psi_a(x,y,z)$ and $\bar{F}=0$ where $\Psi_a(x,y,z)$ is a known scalar quantity.

(f) Let $\bar{F} = \hat{a}_z\left(\Psi_f^+(x,y)\,e^{-jkz} + \Psi_f^-(x,y)\,e^{jkz}\right)$ and $\bar{A}=0$ for TEM waves. Find the ratio of E_x^+/H_y^+ and E_x^-/H_y^-. Note that E_x^+ indicates the positive z-travelling E_x and E_x^- indicates the negative z-travelling E_x. Same for H_y and Ψ_f.