EEE 451 / 551

HW # 8

① A circular waveguide of radius, a, 3 cm that is filled with polystyrene ($\epsilon_r = 2.56$) is used at a frequency of 2 GHz. For the dominant $TE^z_{m,n}$ mode, determine the followings:

(a) Cut-off frequency, (b) Guided wavelength (cm), (c) phase constant, k_z, (rad/cm), (d) wave impedance

② The TE modes of a waveguide can be derived by letting the vector potentials $\bar{A}$ and $\bar{F}$ be equal to $\bar{A} = 0$ and $\bar{F} = \hat{a}_z F_z$. The vector potential $\bar{F}$ must satisfy the vector wave equation which reduces to $\nabla^2 F_z + k^2 F_z = 0$ for $\bar{F} = \hat{a}_z F_z$. Consider a half-circle-shaped cross section for a circular waveguide shown in Fig. 1. The radius of the half circle is 'a'. If one assumes that waves propagate only in the $+z$ direction (assume an ∞'ly long waveguide), the solution for F_z^+ is of the form $F_z^+ = [A_1 J_n(\beta_\rho \rho) + B_1 Y_n(\beta_\rho \rho)][C_2 \cos(n\phi) + D_2 \sin(n\phi)] e^{-j\beta_z z}$

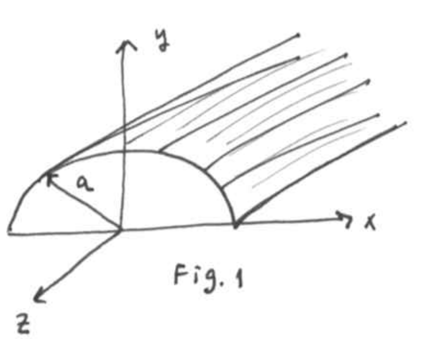


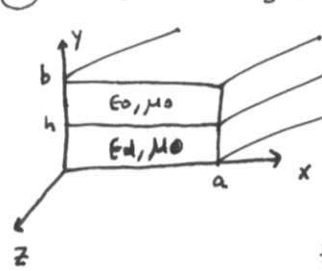
Fig. 1

with $\beta_z^2 + \beta_\rho^2 = k^2$, which is a solution to $(\nabla^2 + k^2) F_z = 0$ in cylindrical coordinates. Derive simplified expressions for the vector potential component, electric and magnetic fields and the cut-off frequencies for TE^z modes.

Hint: Use $\bar{E} = -\frac{1}{\epsilon} \nabla \times \bar{F}$ and $\bar{H} = -j\omega \bar{F} - \frac{j}{\omega\mu\epsilon} \nabla(\nabla \cdot \bar{F})$

BONUS PART for question ②: For problem ② once the vector potential $\bar{F}$ is found, $\bar{H}$ can be found using either $\bar{H} = -j\omega\bar{F} - \frac{j}{\omega\mu\epsilon}\nabla(\nabla\cdot\bar{F})$ or $\bar{H} = \frac{-1}{j\omega\mu}\nabla\times\bar{E}$ with $\bar{E} = -\frac{1}{\epsilon}\nabla\times\bar{F}$. However, at a first glance the resultant $\bar{H}$ fields seem different. Show that $\bar{H}$ fields found using either formulation is correct and they are identical.

③ Consider the geometry of the partially dielectric loaded rectangular waveguide shown in Fig. 2. For such a structure (i.e., this one and similar ones) TE^z and TM^z modes can not satisfy the boundary conditions and hence, a combination of them, so called longitudinal section electric (LSE) or longitudinal section magnetic (LSM) modes, are used. For TE modes (i.e., LSEY or TE^y or H^y modes), assume that the structure is infinitely long along the z-axis and we have only $+z$ travelling waves. To analyze such a geometry, follow the procedure given below and answer the questions asked at each item.



(a) Assume $\bar{F} = F_y^+(x,y,z)\hat{a}_y$ and $\bar{A} = 0$, and solve the Helmholtz equation for F_y in both regions: namely, one for the dielectric region $(0 \le x \le a,\ 0 \le y \le h,\ z)$ and the other for the free-space region $(0 \le x \le a,\ h \le y \le b,\ z)$.

(b) Find the field expressions from $\bar{E} = -\frac{1}{\epsilon}\nabla\times\bar{F}$ & $\bar{H} = -j\omega\bar{F} - j\frac{1}{\omega\mu\epsilon}\nabla(\nabla\cdot\bar{F})$

(c) Apply all boundary conditions and obtain all necessary constants, etc.

(d) Finally, obtain the two transcendental equations.