Solutions to HW#9

① $Z_0 = 50\Omega$ start with an initial guess of $\frac{W}{d} > 2 \Rightarrow B = \frac{377\pi}{2 Z_0 \sqrt{\epsilon_r}} = 7.985$

then $\frac{W}{d} = \frac{2}{\pi}\left[B - 1 - \ln(2B-1) + \frac{\epsilon_r - 1}{2\epsilon_r}\left\{\ln(B-1) + 0.39 - \frac{0.61}{\epsilon_r}\right\}\right] = 3.081$

so $\frac{W}{d} > 2$ (If $\frac{W}{d}$ turns out to be less than 2, you should use the other expression)

$\Rightarrow \frac{W}{d} = 3.081 \Rightarrow \boxed{W = 3.081 \times d = 0.391\,cm}$

$\epsilon_e = \frac{\epsilon_r + 1}{2} + \frac{\epsilon_r - 1}{2}\frac{1}{\sqrt{1 + 12d/W}} \Rightarrow \epsilon_e = 1.87$. Then $\phi = \beta l = k_0\sqrt{\epsilon_e}\, l = \frac{2\pi f}{c} l \sqrt{1.87} = 90°$

$\Rightarrow \boxed{l = 2.19\,cm}$

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② $\beta_z = \beta = \sqrt{k_0^2 - k_c^2} = \sqrt{\left(\frac{\omega}{c}\right)^2 - k_c^2} \Rightarrow \frac{d\beta}{d\omega} = \frac{\omega/c^2}{\sqrt{\left(\frac{\omega}{c}\right)^2 - k_c^2}} = \frac{k_0}{c\beta}$

$\Rightarrow \boxed{v_g = \left(\frac{d\beta}{d\omega}\right)^{-1} = \frac{c\beta}{k_0}}$, $\boxed{v_p = \frac{\omega}{\beta} = \frac{k_0 c}{\beta}}$

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③ $\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix}\begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$ and $\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix}\begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$

Z-parameters:

$Z_{11} = \frac{V_1}{I_1}\Big|_{I_2=0} \Rightarrow Z_{11} = Z_0 \frac{Z_L + jZ_0 \tan\beta l}{Z_0 + jZ_L \tan\beta l}\Big|_{Z_L=\infty} \Rightarrow \boxed{Z_{11} = -jZ_0 \cot\beta l}$, $\boxed{Z_{22} = Z_{11} = -jZ_0\cot\beta l}$

$Z_{12} = \frac{V_1}{I_2}\Big|_{I_1=0} \Rightarrow V(z) = V^+ e^{-j\beta z} + V^- e^{j\beta z}$

$I(z) = \frac{V^+}{Z_0} e^{-j\beta z} - \frac{V^-}{Z_0} e^{j\beta z} \Rightarrow I(0) = I_1 = 0 = \frac{V^+}{Z_0} - \frac{V^-}{Z_0} \Rightarrow \underline{V^+ = V^-}$

$I(z = l) = \frac{V^+}{Z_0}\left(e^{-j\beta l} - e^{j\beta l}\right) = -I_2 \Rightarrow V^+ = \frac{-I_2 Z_0}{(e^{-j\beta l} - e^{j\beta l})} \Rightarrow V^+ = \frac{I_2 Z_0}{j2\sin\beta l}$

$V(z=0) = V_1 = 2V^+ = \frac{2 I_2 Z_0}{2j\sin\beta l}$

$\Rightarrow \frac{V_1}{I_2}\Big|_{I_1=0} = \frac{2I_2 Z_0 / 2j\sin\beta l}{I_2} \Rightarrow \boxed{Z_{12} = \frac{Z_0}{j\sin\beta l}}$ $\boxed{Z_{21} = Z_{12} = \frac{Z_0}{j\sin\beta l}}$

Y-parameters:

$\boxed{Y_{11} = \frac{I_1}{V_1}\Big|_{V_2=0} = -jY_0\cot\beta l = Y_{22}}$

$Y_{12} = \frac{I_1}{V_2}\Big|_{V_1=0}$ $V(z=0) = V_1 = 0 \Rightarrow V^+ = -V^-$

$I(z=0) = I_1 = \frac{2V^+}{Z_0} \Rightarrow V^+ = \frac{Z_0 I_1}{2}$

$V(z=l) = V_2 = V^+\left(e^{-j\beta l} - e^{j\beta l}\right)$

$\Rightarrow V_2 = \frac{Z_0 I_1}{2}\left(-2j\sin\beta l\right)$

$$\Rightarrow Y_{12} = Y_{21} = \frac{I_1}{\frac{Z_0 I_1}{2}(-2j \sin\beta l)} \Rightarrow \boxed{\frac{j Y_0}{\sin\beta l} = Y_{12} = Y_{21}}$$ ②

where $Y_0 = \frac{1}{Z_0}$

Note that ① $Z_{in} = Z_0 \frac{Z_L + jZ_0 \tan\beta l}{Z_0 + jZ_L \tan\beta l}$, $Y_{in} = \frac{1}{Z_{in}} = Y_0 \frac{Z_0 + jZ_L \tan\beta l}{Z_L + jZ_0 \tan\beta l} = Y_0 \frac{\frac{1}{Y_0} + j\frac{1}{Y_L}\tan\beta l}{\frac{1}{Y_L} + j\frac{1}{Y_0}\tan\beta l}$

$$\Rightarrow Y_{in} = Y_0 \frac{Y_L + jY_0 \tan\beta l}{Y_0 + jY_L \tan\beta l}.$$

③ When the Z & Y parameters of a transmission line segment is investigated, we notice that to obtain Y parameters, one can use the expressions for the Z-parameters (same equations apply) by replacing Z_{11} by Y_{11} and Z_0 by Y_0, etc.

④ $Z_0 = 1$ and using the results of ③

(a) $Z_{11} = Z_1 - j\cot\beta l$

$Z_{22} = Z_2 - j\cot\beta l$

$Z_{12} = \frac{1}{j\sin\beta l}$

(b) if $\beta l = \frac{\pi}{2}$ $\Rightarrow$ $Z_{11} = Z_1$

$Z_{22} = Z_2$

$Z_{12} = -j$