Explicit Descriptive Equations to Construct Symmetrical Lossless Two Ports with Mixed Lumped and Distributed Elements

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Abstract

In this work, explicit design equations to describe symmetrical low-pass ladders (SLPL) constructed with mixed-lumped and distributed elements are presented. It is expected that the formulas introduced in this paper will be useful to design microwave broadband matching networks and amplifiers for wireless communication systems. It is also hopped that high frequency interconnects can be modelled utilizing the explicit equations presented in this paper.

1. Introduction

In microwave circuit design, use of mixed lumped and distributed elements presents several advantages.

Mixed element design problem has been investigated extensively in the literature [7,9]. Recently based on a multivariable approach, an efficient semi-analytic procedure is proposed by Aksen and Yarman [1,2,8,10] to construct some regular mixed lumped-distributed structures.

In this study, two-variable scattering description of symmetrical mixed type structures is investigated. In particular, for the low-pass symmetrical mixed ladder forms shown in Figure 1 explicit design equations are obtained.

2. Scattering Description of Symmetrical Mixed Element Two-ports

Symmetrical lossless two-ports constructed with two kinds of elements may be described in terms of scattering parameters $S_{ij}(p,\lambda)$; i=1,2 where $p=\sigma+jw$ is associated with lumped elements and $\lambda=\tanh(p\tau)$ is associated with distributed elements.

Using Belevitch representation the scattering parameters of a symmetrical lossless two-port is given by,

$$S_{11}=\frac{h(p,\lambda)}{g(p,\lambda)} \qquad S_{12}=\sigma\frac{f(-p,-\lambda)}{g(p,\lambda)}$$
$$S_{21}=\frac{f(p,\lambda)}{g(p,\lambda)} \qquad S_{22}=-\sigma\frac{h(-p,-\lambda)}{g(p,\lambda)} \tag{1}$$

where the two variable polynomials can be expressed in the form

$$h(p,\lambda)=p^T\Lambda_H\lambda \qquad g(p,\lambda)=p^T\Lambda_G\lambda$$
$$f(p,\lambda)=p^k\left(1-\lambda^2\right)^{n_\lambda/2}$$
$$p^T=[1\ p\ p^2 \ldots p^n], \qquad \lambda=[1\ \lambda\ \lambda^2 \ldots \lambda^n]. \tag{2}$$

In the above formulation n_p stands for the total number of lumped circuit elements, n_λ designates the total number of unit elements which connects the lumped element in the cascade form. The polynomials $h(p,\lambda)$ and $g(p,\lambda)$ are fully described by the connectivity matrices Λ_H and Λ_G which are formed by the polynomial coefficients.

For the particular case of low-pass symmetrical ladder networks shown in Figure 1(a-c), k is selected to be zero in the polynomial $f(p,\lambda)$. For this case, the generic form of $\Lambda_H=[h_{ij}]$ matrix is given by

$$\Lambda_H=\begin{bmatrix} 0 & h_{01} & 0 & \ldots & h_{0n_\lambda} \\ h_{10} & 0 & h_{12} & \ldots & h_{1n_\lambda} \\ 0 & h_{21} & 0 & \cdots & h_{2n_\lambda} \\ h_{30} & 0 & h_{32} & \cdots & h_{3n_\lambda} \\ 0 & h_{41} & 0 & \cdots & h_{4n_\lambda} \\ 0 & 0 & h_{52} & \cdots & h_{5n_\lambda} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ h_{n_p0} & h_{n_p1} & h_{n_p2} & \cdots & h_{n_pn_\lambda} \end{bmatrix} \tag{3}$$

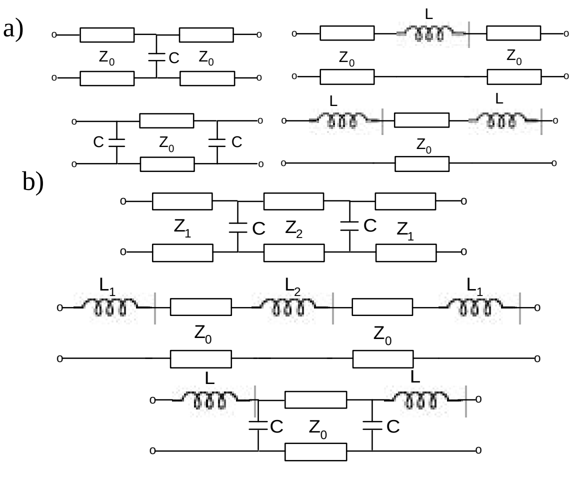

Fig. 1 Some typical symmetrical networks
a) 3-elements b)5 elements

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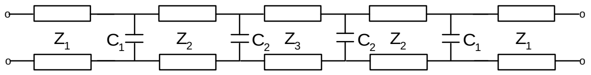

Fig. 1 c Typical symmetrical network with 9 elements

Table 1   3 element networks

For n_p=2, n_λ=1 (C-UE-C)

$$\Lambda_H = \begin{bmatrix} 0 & \frac{Z_0}{2} - \frac{1}{2Z_0} \\ -C & 0 \\ 0 & -\frac{1}{2} Z_0 C^2 \end{bmatrix} \quad \Lambda_G = \begin{bmatrix} 1 & \frac{Z_0}{2} + \frac{1}{2Z_0} \\ C & CZ_0 \\ 0 & \frac{1}{2} Z_0 C^2 \end{bmatrix}$$

$h_{10} = -C$ $\qquad h_{10} < 0$

$h_{21} = -\frac{1}{2} Z_0 C^2$ $\qquad h_{21} < 0$

Free parameters

For n_p=2, n_λ=1 (L-UE-L)

$$\Lambda_H = \begin{bmatrix} 0 & \frac{Z_0}{2} - \frac{1}{2Z_0} \\ L & 0 \\ 0 & \frac{1}{2} \frac{L^2}{Z_0} \end{bmatrix} \quad \Lambda_G = \begin{bmatrix} 1 & \frac{Z_0}{2} + \frac{1}{2Z_0} \\ L & \frac{L}{Z_0} \\ 0 & \frac{1}{2} \frac{L^2}{Z_0} \end{bmatrix}$$

$h_{10} = L$ $\qquad h_{10} > 0$

$h_{21} = \frac{1}{2} \frac{L^2}{Z_0}$ $\qquad h_{21} > 0$

Free parameters

For n_p=1, n_λ=2 (UE-C-UE)

$$\Lambda_H = \begin{bmatrix} 0 & Z_0 - \frac{1}{Z_0} & 0 \\ -\frac{1}{2} C & 0 & \frac{1}{2} C Z_0^2 \end{bmatrix} \Lambda_G = \begin{bmatrix} 1 & Z_0 + \frac{1}{Z_0} & 1 \\ \frac{1}{2} C & Z_0 C & \frac{1}{2} C Z_0^2 \end{bmatrix}$$

Free parameters

$h_{10} = -\frac{1}{2} C$ $\qquad h_{10} < 0$ $\qquad h_{12} = \frac{1}{2} C Z_0^2$ $\qquad h_{12} > 0$

For n_p=1, n_λ=2 (UE-L-UE)

$$\Lambda_H = \begin{bmatrix} 0 & Z_0 - \frac{1}{Z_0} & 0 \\ \frac{1}{2} L & 0 & -\frac{1}{2} \frac{L}{Z_0^2} \end{bmatrix} \Lambda_G = \begin{bmatrix} 1 & Z_0 + \frac{1}{Z_0} & 1 \\ \frac{1}{2} L & \frac{L}{Z_0} & \frac{1}{2} \frac{L}{Z_0^2} \end{bmatrix}$$

Free parameters

$h_{10} = \frac{1}{2} L$ $\qquad h_{10} > 0$

$h_{12} = -\frac{1}{2} \frac{L}{Z_0^2}$ $\qquad h_{12} < 0$

Referring to Figure 1a-c, descriptive scattering parameters of 3 element, 5 element and 9 element typical symmetrical sections are given in Table 1, Table 2, and Table 3 respectively in terms of the connectivity matrix Λ_H and Λ_G. Here it should be noted that some of the entries of Λ_H and Λ_G matrices can be expressed in terms of an independent set of elements in Λ_H matrix. That is, the dependent elements are expressed in terms of freely chosen coefficients of h_{ij} in the corresponding tables. In this way, by means of the free coefficients, Λ_H and Λ_G matrices, and hence the scattering parameters of the selected symmetrical network topologies are obtained.

Therefore, any design problem or interconnect modelling problem reduces to the determination of the independently selected elements of the connectivity matrix of Λ_H.

Table 2 5 element network

For n_p=3, n_λ=2 (UE-C-UE-C-UE)

$$\Lambda_H = \begin{bmatrix} 0 & Z_1 - \frac{1}{Z_1} + \frac{1}{2} Z_2 - \frac{1}{2Z_2} & 0 & \frac{1}{2}\frac{Z_1^2}{Z_2} - \frac{1}{2}\frac{Z_2}{Z_1^2} \\ -C & 0 & Z_1^2 C + Z_1 Z_2 C - \frac{Z_2}{Z_1} C & 0 \\ 0 & -\frac{1}{2} Z_2 C^2 & 0 & \frac{1}{2} Z_1^2 Z_2 C^2 \end{bmatrix}$$

$$\Lambda_G = \begin{bmatrix} 1 & Z_1 + \frac{1}{Z_1} + \frac{1}{2} Z_2 + \frac{1}{2Z_2} & \frac{Z_1}{Z_2} + \frac{Z_2}{Z_1} + 1 & \frac{1}{2}\frac{Z_1^2}{Z_2} + \frac{1}{2}\frac{Z_2}{Z_1^2} \\ C & 2CZ_1 + CZ_2 & Z_1^2 C + Z_1 Z_2 C + \frac{Z_2}{Z_1} C & Z_2 C \\ 0 & \frac{1}{2} Z_2 C^2 & Z_1 Z_2 C^2 & \frac{1}{2} Z_1^2 Z_2 C^2 \end{bmatrix}$$

Free parameters

$h_{10} = -C$ $\qquad h_{10} < 0$

$h_{21} = -\frac{1}{2} Z_2 C^2$ $\qquad h_{21} < 0$

$h_{23} = \frac{1}{2} Z_1^2 Z_2 C^2$ $\quad h_{23} > 0$

3. Application of Explicit Formulas in Design Problems

Once the circuit topology is selected for the design of broadband matching network or microwave amplifiers, the designer can immediately determine the selected independent parameters to optimize the transducer power gain of the system. In the core of the design we propose the following algorithm.

Algorithm:

Design of symmetrical low-pass ladder network constructed with mixed lumped and distributed element for broadband matching problem:

Step 1: Select the appropriate circuit topology to be implemented on MMICs.

Step 2: Express all the connectivity matrices $\Lambda_H=[h_{ij}]$ and $\Lambda_G=[g_{ij}]$ in terms of the independently chosen parameter h_{kl} of the Λ_H matrix.

Step 3: Express the transducer power gain of the system in terms of freely chosen parameters h_{kl}.

Step 4: By means of an optimization package, determine the freely chosen parameter to maximize the transducer power gain of the frequency band of operation.

<u>Remarks:</u>

1. For a typical double matching problem TPG as given by

$$T(p,\lambda)=\frac{(1-|S_G|^2)(1-|S_L|^2)|f(p,\lambda)|^2}{|g(p,\lambda)-h(p,\lambda)S_G+óS_L h(\text{-}p,\text{-}\lambda)-S_G g(p,\lambda)|^2}$$

2. Obviously TPG is non-linear in terms of h_{kl}. However, initial given for h_{kl} can be determined as described by [4,5,6]

3. The above algorithm may as well be utilize to model interconnects. In this case the objective function of the optimization package must be described in terms of the measured scattering parameters of the interconnect under consideration.

For this application, the objective function ε may be written as

$$\varepsilon=\sum_{r=1}^{N}\left|S_{11m}(jw_r)-S_{11}(jw_r,j\Omega_r)\right|$$

where $S_{11}(jw_r)$ designates the measured input reflection coefficient of the interconnect over the frequencies w_r and $S_{11}(jw_r,j\Omega_r)$ designates the models unit normalized scattering coefficient for $p=jw_r$, $\lambda=j\Omega_r$, $\Omega_r=\tan(w_r\tau)$. τ is the constant delay length of the unit elements which is picked by the designer.

Table 3 9 element network

For $n_p=4$, $n_\lambda=5$ (UE-C-UE-C-UE-C-UE-C-UE)

$$h_{03}=-\frac{Z_2}{Z_1Z_3}-\frac{Z_3}{2Z_2^2}-\frac{Z_3}{2Z_1^2}+\frac{Z_2^2}{2Z_3}-\frac{1}{Z_1}+\frac{Z_1Z_2}{Z_3}+\frac{Z_1^2}{Z_2}-\frac{Z_2}{Z_1^2}+\frac{Z_1^2}{2Z_3}+\frac{Z_1Z_3}{Z_2}-\frac{Z_3}{Z_1Z_2}+Z$$

$$h_{12}=-C_1\frac{Z_2}{Z_3}+2C_2Z_1Z_2+2C_1Z_1Z_2+C_1Z_1^2+C_2Z_2^2+C_2\frac{Z_1}{Z_2}+C_1\frac{Z_1}{Z_3}-C_1\frac{Z_3}{Z_2}-2C_1\frac{Z_2}{Z_1}-C_1-C_1\frac{Z_3}{Z_1}$$
$$-C_2\frac{Z_3}{Z_2}-C_2\frac{Z_3}{Z_1}+C_2Z_2Z_3-2C_2\frac{Z_2}{Z_1}+C_2Z_2^2$$

$$h_{14}=C_1\frac{Z_1^2Z_3}{Z_2}+C_1\frac{Z_1Z_2^2}{Z_3}+C_1Z_1^2+C_2\frac{Z_1^2Z_3}{Z_2}+C_2Z_1Z_3-C_1\frac{Z_2^2}{Z_1Z_3}+C_1\frac{Z_1^2Z_2}{Z_3}-C_2\frac{Z_2Z_3}{Z_1^2}-C_2\frac{Z_3}{Z_1}-C_2\frac{Z_2^2}{Z_1^2}$$

$$h_{21}=-C_1C_2Z_3-2C_1C_2Z_2-\frac{1}{2}C_1^2Z_3-\frac{1}{2}C_2^2Z_3-C_1^2Z_2$$

$$h_{23}=-C_1C_2Z_3+C_1C_2Z_1^2Z_3+2C_1C_2Z_1Z_2Z_3+\frac{1}{2}C_2^2Z_2^2Z_3+2C_1C_2Z_1Z_2^2-2C_1C_2\frac{Z_2Z_3}{Z_1}+2C_1C_2Z_1^2Z_2+\frac{1}{2}C_1^2Z_1^2Z_3$$
$$+C_2^2Z_1Z_2Z_3+C_1^2Z_1^2Z_2+\frac{1}{2}C_2^2Z_1^2Z_3-C_2^2\frac{Z_2Z_3}{Z_1}-2C_1C_2\frac{Z_2^2}{Z_1}-\frac{1}{2}C_1^2\frac{Z_2^2}{Z_3}$$

$$h_{25}=C_1C_2Z_1^2Z_3+C_1^2\frac{Z_1^2Z_2^2}{Z_3}-C_2^2\frac{Z_2^2Z_3}{Z_1^2}$$

$$h_{32}=-C_1^2C_2Z_1Z_2-C_1C_2^2Z_1Z_2-C_1^2C_2Z_1^2$$

$$h_{34}=C_1C_2^2Z_1Z_2^2Z_3+C_1^2C_2Z_1^2Z_2^2-C_1C_2^2\frac{Z_2^2Z_3}{Z_1}+C_1^2C_2Z_1^2Z_2Z_3+C_1C_2^2Z_1^2Z_2Z_3$$

$h_{00}=h_{02}=h_{04}=0$ $h_{11}=h_{13}=h_{15}=0$ $h_{20}=h_{22}=h_{24}=0$

$h_{30}=h_{31}=h_{33}=h_{35}=0$ $h_{40}=h_{41}=h_{42}=h_{44}=0$

Free parameters

$$h_{01}=Z_1+Z_2+\frac{1}{2}Z_3-\frac{1}{Z_1}-\frac{1}{Z_2}-\frac{1}{2Z_3}$$

$h_{05}=\frac{1}{2}\frac{Z_1^2Z_3}{Z_2^2}-\frac{1}{2}\frac{Z_2^2}{Z_1^2Z_3}$ $h_{10}=-C_1-C_2$ $h_{10}<0$

$h_{43}=-\frac{1}{2}C_1^2C_2^2Z_{21}^2Z_2$ $h_{43}<0$ $h_{45}=\frac{1}{2}C_1^2C_2^2Z_1^2Z_2^2Z_3$ $h_{45}>0$

4. Example

In this example we wish to solve the double matching problem depicted in Figure 2. For the given load and generator impedances it is desired to design a symmetrical equalizer of order 5. For this case, using the explicit expressions given in Table 2 the TPG is optimized over $0 \le w \le 1$ and free parameters are chosen as h_{10}, h_{21} and h_{23}. As a result of optimization the following form of the descriptive polynomials are obtained.

$$h(p,\lambda) = p^T \begin{bmatrix} 0 & 2.5451 & 0 \\ -1.3297 & 0 & -0.9141 \\ 0 & -5.1059 & 0 \\ 0 & 0 & -0.6132 \end{bmatrix} \lambda$$

$$g(p,\lambda) = p^T \begin{bmatrix} 1 & 3.2369 & 1 \\ 1.3297 & 7.6883 & 1.6565 \\ 0 & 5.1059 & 0.9542 \\ 0 & 0 & 0.6132 \end{bmatrix} \lambda$$

Final transducer power gain is depicted in Figure 3 and the synthesis result is given in Figure 2.

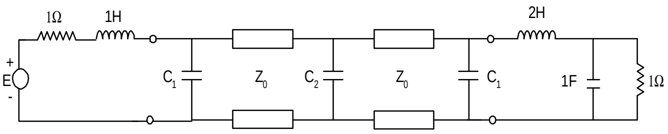

Fig. 2 Double matching problem (Z_0=2.8910, C_1=1.2852, C_2=0.0888)

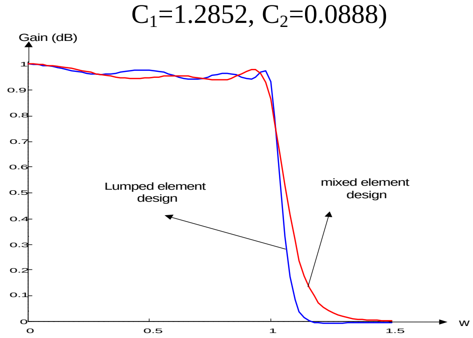

Fig. 3 Gain response of the double matching structure

The resulting performance of the matching network design with symmetrical mixed lumped-distributed elements is in close agreement with the available solutions in the literature employing only lumped elements [1,5].

4. References:

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